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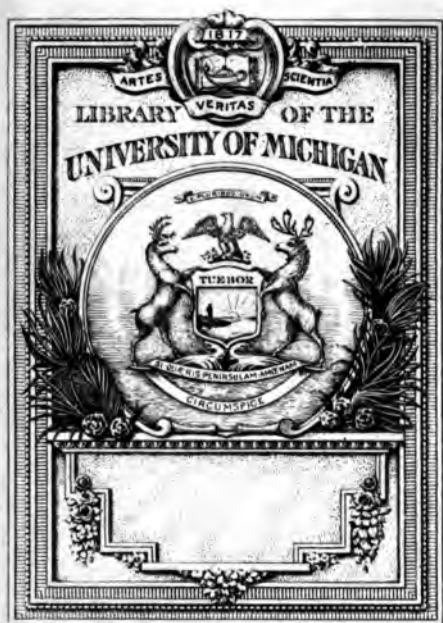
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THE
CALCULATION
OF

Solar Eclipses

WITHOUT
PARALLAXES.





THE
CALCULATION
OF
Solar Eclipses
WITHOUT
PARALLAXES.

WITH
A SPECIMEN of the same in the
Total Eclipse of the Sun, *May 11. 1724.*

Now first made Publick.

To which is added,

A PROPOSAL how, with the Latitude
given, the Geographical Longitude of all the
Parts of the Earth may be settled by the bare
Knowledge of the Duration of Solar Eclipses,
and especially of Total Darknes.

WITH

An ACCOUNT of some late Observations made with
Dipping Needles, in order to discover the LONGITUDE
and LATITUDE at Sea.

HC

By *WILL. WHISTON*, M. A.
Sometime Professor of the Mathematicks in the
University of *Cambridge*.

L O N D O N:

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LEMMA TA:

O R,

Preparatory Propositions.

I.



THE most useful and most remarkable Cycle or Period for the Revolution of Eclipses, both Solar and Lunar, mentioned by *Pliny*, (*Nat. Hist.* II.) and by him only of all the Ancients, is the Interval of 223 Synodical Months = 6585 Days : or = 18 *Julian* Years: with 10 Days, when the Cycle or Period contains 5 Leap-Years : and with 11 Days, when with 7 Hours 43 $\frac{1}{4}$. In which Time the direct mean Motion of the *Moon* and her *Apogee* in the Ecliptick, is nearly so much *more* ; and the Retrograde mean Motion of the *Nodes* nearly so much *less* than entire Révolutions, as the mean Motion of the *Moon* from the *Sun*, upon which all mean

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Conjunctions, Oppositions, and Eclipses properly depend, exceeds the like entire Revolutions. Which Coincidences do therefore nearly restore the mean State of the Moon it self, its Apogee, Nodes, and Lunations : And produce an eminent Revolution of correspondent New Moons, Full Moons, and Eclipses, after that Interval perpetually.

This appears by the following Calculation of all these mean Motions from the Astronomical Tables.

Mean Motion of the Sun or Earth.

		s	o	'	"
Years	18	11	29	28	32
Days	11	o	10	50	32
Hours	7	o	o	17	15
'	43	o	o	1	46
"	15	o	o	o	1
Sum		o	10	48	6

Mean Motion of the Moon in the Ecliptick.

		s	o	'	"
Years	18	7	11	37	22
Days	11	4	24	56	25
Hours	7	o	03	50	35
'	43	o	00	23	36
"	15	00	00	00	08
Sum		1	00	10	48

Mean

[3]

Mean Motion of the Apogee.

		s	o	i	"
Years	18	o	12	23	53
Days	11	o	1	13	32
Hours	7	o	o	1	57
	' 43 $\frac{1}{4}$	o	o	o	12
Sum			o	13	39 34

Mean Motion of the Node backward.

		s	o	i	"
Years	18	11	18	7	39
Days	11	o	o	34	57
Hours	7	o	o	o	56
	' 43 $\frac{1}{4}$	o	o	o	6
Sum			11	18	43 38

Mean Motion of the Moon from the Sun.

		s	o	i	"
Years	18	7	11	58	50
Days	11	4	14	5	53
Hours	7	o	3	33	20
	' 43	o	o	21	50
	" 15	o	o	o	7
Sum			o	o	o o

	o 1 11
From the mean Motion of the Apogee—	13 39 34
Subſtraſt that of the Moon in the Eclipt. -	10 48 6
Remains ——— (2° 51' 28") ———	2 51 28

To the mean Motion of the Moon--	o 10 48 6
Add that of the Node ———	11 18 43 38
Sum ———	11 29 31 44
Difference from 12 Signs (0° 47')—	o o 28 16

Whence it appears that the Difference of the mean Motions of the Apogee and of the Nodes from that of the Moon her ſelf in the Ecliptick, in ſuch a Period, is but ſmall : Not more in the former Caſe than $2^{\circ} 51' 28'' = 2^{\circ} 51'$, nor in the latter than $28' 16'' = 0^{\circ} 47'$. Whence alſo it appears that the Lunar Apogee does in every ſuch Cycle differ but $\frac{1}{12}$ of the entire Difference $2^{\circ} 51' 28''$ (62). That the Lunar Node only differs in that Cycle $\frac{1}{12}$ of the entire Difference $0^{\circ} 47'$ (383). And that the Anomaly of the Sun it ſelf differs only $\frac{1}{12}$ of the entire Anomaly : $10^{\circ} 8'$ (166). Which Quantities being generally ſmall, cannot occaſion any great Inequality in the Times and Circumſtances of New and Full Moons, or of Eclipſes ; nor by conſequence greatly diſturb the regular Succeſſion of the ſame in any ſingle Period ; nor indeed very greatly in ſeveral ſucceſſive Periods. For ſince the mean Motion of the *Moon from the Sun* is within a very ſmall Matter ever certain and invariable, that Revolution is always juſt ; and always determines the mean Time of all Conjunctions, Oppoſitions, and Eclipſes rightly ; and ſince the other Anomalies are but ſmall, and always come right again in Length of Time, they cannot ever produce any

very

very great Anomalies in our Calculations from them. As will farther appear under the following *Scholia*.

N. B. This Period for Eclipses, has of late been called, both by Mr. *Flamsteed*, and Dr. *Halley*, the *Saros*, or the *Chaldean Saros*: As if it were known and us'd by the old *Chaldeans*, and thence called by that Name. For which I know no sufficient Foundation. There is indeed a gross Mistake of *Pliny's* Number in *Suidas*, (who thus applies this term) 222 for 223 Months, as almost all the Editions of *Pliny* still have it; and He calls that Period by this *Chaldean* Name *Saros*. Yet the *Chaldeans* never, that we find, apply'd it to any other Period than that of 3600 *Years* or *Days*; by which Period alone all the Antediluvian Reigns are determined both in *Abydenus* and *Berosus* themselves, from the ancientest Records of that Kingdom. See my *New Theory*, the later Editions, *Hypoth. X. Lem.* to the third Argument; and *Appendix* to the *Essay* towards restoring the true Text of the Old Testament, p. 203, — 213.

SCHOLIA.

(1.) We may here Observe, that since the Limit for Eclipses of the Moon is about $11^{\circ} 40' = 700'$ on each Side of the Node; as is the Limit for Eclipses of the Sun, about $16^{\circ} 40' = 1000'$. If we divide 700, the Limit of the Moon's Eclipses, by $28^{\circ} 16'' = 28\frac{1}{3}$, which is the Difference between the Revolution of the Moon to the Sun and of the Node above given, we shall have nearly Twenty Five for the Number of Cycles,, after a Central Lunar Eclipse in one
of

of the Nodes, before the Moon goes off the shadow of the Earth entirely at the same Node, and 450 Years ($25 \times 18 = 450$), or double that Number 900 Years for the Time that the Moon begins to enter the Ecliptick Limit on one Side, till it goes out of it on the other. During which long Interval there will still be Eclipses of the Moon each Period. And if we divide 1000' the Limit of the Sun's Eclipses, by the same Number $28\frac{1}{3}$, we shall have nearly 35 for the Number of Periods after a Solar Central Eclipse at the Middle of the Earth, in one of the Nodes, before the Penumbra goes off the *North* or *South* Parts of the Disk of the Earth entirely at the same Time; *i. e.* 630 Years. ($35 \times 18 = 630$), or double that Number 1260 Years, from the Time that the Moon in any such Period begins to enter the Ecliptick Limit on one Side, till it goes out of on the other: During which longer Interval there will still be somewhere Eclipses of the Sun each Period. After which respective long Intervals of Time there will be no such Eclipses for much longer Intervals.

(2.) Since the utmost Latitude of the Moon that can permit any Lunar Eclipse, is about $62'$, and the same utmost Latitude that can permit a Solar Eclipse is about $92'$: If we divide the first Number by 25, or the last by 35, the Numbers of Revolutions for the Ecliptick Limits, we shall have about $2\frac{1}{6} = 2':36''$ for the mean Alteration of the Moon's Latitude in each single Period all along; and this both for Solar and Lunar Eclipses. Which Latitude will be *South* during the one half of the long Period of the Ecliptick Limits before-mentioned; and *North* during the other half: Gradually increasing,
and

and as gradually decreasing perpetually: As in the following Table.

A Table of the mean Latitudes of the Moon each single Cycle, either *North* or *South*; beginning at an Eclipse in one of the Nodes, without any Latitude at all.

Latitudes for 25 Cycles in Lunar, and 35 in Solar Eclipses.

Cycles ' "

1	2	26
2	4	52
3	7	19
4	9	46
5	12	14
6	14	43
7	17	12
8	19	42
9	22	12
10	24	43
11	27	14
12	29	46
13	32	19
14	34	52
15	37	26
16	40	1
17	42	36
18	45	12
19	47	48
20	50	25
21	53	3
22	55	41
23	59	20
24	60	59
25	63	39
26	66	0
27	68	41
28	71	23
29	74	6
30	76	49
31	79	33
32	82	17
33	85	2
34	87	48
35	90	34

(3.) Since

(3.) Since the principal Alteration in the Quantity and Duration of total Eclipses of the Sun, arises from the Difference there is at any Time between the real Distances, and apparent Diameters of the Sun and Moon, at the Time of such Eclipses, that Quantity and Duration must depend on the Difference of their mean Anomalies, which gives us that Difference of Distances and Diameters ; and must therefore answer in each Cycle one with another, to the Differences of those mean Anomalies during that Interval ; which in the Sun comes to $\frac{100.8}{180}$ or $\frac{1}{16.6}$ of its entire Anomaly. And in the Moon to $\frac{1}{11.2}$ or $\frac{1}{11.2}$ of its entire Anomaly. And since the whole mean Excentricity of the Moon is somewhat above three Times as great as that of the Sun, or as $\frac{1}{11.2}$ to $\frac{1}{11.2}$. The Differences of the Sun's Distances and Diameters will be but a little greater in each Period one with another, than those of the Moon.

(4.) When therefore the Anomalies of the Sun and Moon are of the same Species ; I mean both ascending, or both descending ; their Distances and Diameters will, one with another, increase or decrease nearly in the same Proportion ; and the Quantity and Duration of total Darkness will alter but little in such a Period. But when those Anomalies are of the contrary Species ; that is, the one ascending while the other descends ; they will alter considerably. So that if the Sun be descending, and its apparent Diameter Increasing ; while the Moon is ascending, and its apparent Diameter Decreasing, the Eclipse of the Sun will, each succeeding Cycle, afford a smaller total Shadow ; till at last it afford no total Shadow

dow at all ; but the Eclipses become *Annular*. And if the Sun be Ascending while the Moon is Descending, the contrary will happen ; and the total Shadow grow greater perpetually. From which Circumstances of the Sun and Moon in each Revolution of the Cycle duly considered , we may nearly determine whether any succeeding correspondent Eclipse will afford us a greater or less total Shadow, or whether the Eclipses will be only Annular.

(5.) From the like Circumstances we may also nearly determine whether such Eclipses will come somewhat sooner or later , than that of the mean Revolution of the Period before us. For if the Earth be much nearer its Aphelion , than the Moon its Apogæon, at the end of any Cycle , and by consequence if the Earth then revolve comparatively slower , and the Moon swifter than ordinary ; the meeting of the Luminaries will be accelerated. And if the Earth be much remoter from its Aphelion than the Moon from its Apogæon, the contrary will happen ; and the Moon will be later than ordinary e'er it overtake the Sun. So that in the former Case the Eclipse will come a little *before*, and in the latter a little *after* the proper Conclusion of that Period.

(6.) Since the Motion of the Node backward in one of these Periods does not quite reach to the Conjunction or Opposition, that Node must every Cycle go forward, with respect to the Lunations and Eclipses ; and at the ascending Node the Moon will pass more *Southward*, and at the descending Node more *Northward* successively. Thus at the Solar Eclipse May 1. 1706. the Moon, near its ascending Node, had greater *Northern* Latitude than it will have at the next corresponding Solar Eclipse, May 11. 1724. And thus at
C the

the total Solar Eclipse, *April 22. 1715.* the Moon near its descending Node, had less *Northern* Latitude than it will have at its corresponding great Eclipse; *May 2. 1733.*

(7.) Since the Motion of the Moon's Apogee forward is greater in one of these Periods than that of the Lunations, that Apogee must also go forward every Cycle: And if at any one Solar Eclipse that Apogee be in quadrature with the Sun, after it had been in Conjunction, the Moon will the next Period descend by going backward in its Eclipses, towards the Perigee. And if at any one such Apogee it be in the quadrature, after it had been in Opposition, it will the next Cycle ascend: The Reverse of all which is true in Lunar Eclipses.

(8.) The Place and Motion of the Sun in its Ellipsis is so easily known, and that for many Ages, by bare Memory and Reflection, that a few Words will suffice. The Sun is now farthest from the Earth Eight Days after the longest Day; and nearest to it Eight Days after the shortest: And its Motion about 1 Degree in 72 Years. Whence it is evident, that it has, for all the past Ages of Astronomy, been about the Summer Solstice in our Apogee, and about the Winter Solstice in our Perigee; if I may use the Terms of the *Ptolemaick* System. Nor is it therefore any Wonder that the greatest total Eclipses of the Sun have happened still in the Summer, and the greatest Annular ones in the Winter half Year: Since the farther the Sun is off in the Winter, the less must be its apparent Diameter, and by consequence the greater the Excess of the Moon's Diameter above it. On which Excess alone the Greatness of such Eclipses depends. And the Reverse is equally evident in the Case of Annular

Iar Eclipses in Winter: Nor indeed is it very strange, that Annular Eclipses are in these Parts of the World so rarely observed; since they most usually happen in Winter Days; which being short, must afford us a proportionably small Number of them. To the Inhabitants of the other Side of the Equator the Reverse must happen. But those Eclipses very rarely come to our Notice.

(9.) Since this Period reaches only from the middle of one general Eclipse to another, without regard to the Position of any particular Place on the Earth's Surface, arising from the diurnal Motion, we must remember that if an Eclipse of the Sun happens at any particular Place considerably before Noon, it will come sooner, and after Noon later than the proper Conclusion of this Period. Though it must be noted, that Eclipses of the Moon being absolute in their own Nature, are here wholly unconcerned; and no way subject to any Acceleration, Retardation, or Alteration on account of the diurnal Motion of the Earth.

(10.) The principal Alteration of the Time of the Day in all Eclipses depends on the Excess of this Period above an even Number of Days, which is 7 Hours and 43 $\frac{1}{2}$. So that the Cycle does naturally put every correspondent Eclipse later than the foregoing, almost 8 Hours, or one third part of a Day; which thing, by reason of the intervening diurnal Motion, greatly alters all Eclipses, especially Solar, not only as to the bare Time of the Day when, but also as to the Places on the Earth where such correspondent Eclipse will be visible.

(11.) If therefore we join three of these Cycles together, those odd Hours and Minutes will amount nearly to a whole Day; and will therefore nearly bring the middle Point of the correspondent Eclipses to the same Time, in the same Place, and, in part, with the same Circumstances as before: Which a single Cycle cannot possibly do. Only with the Anticipation of 56^l in Time. Which three single Cycles therefore of 19,756 Days, or of 54 Years; with 32 or 33 Days; I call the *Grand Cycle*. And this will be, I think, of the greatest and readiest use in remote Eclipses of any other Period whatsoever. Thus, for Example, there was a total Eclipse of the Sun on *Black Monday*; as it has thence been called ever since; *March 29. 1652.* about Ten a Clock in the Morning. Total, I say it was in the North of *Ireland*, and the *Northwest* of *Scotland*, tho' not so at *London*, or the remoter Parts of *England* and *Scotland*. To this if we add one Cycle, the Time of the next correspondent Eclipse will thus be discovered:

	y	d	h	m	sec
To A. D. 1652. <i>March</i>	00	28	22	00	00
Add	18	10	7	43	15
Sum 1670 <i>April</i>	00	8	5	43	15

So that the correspondent total Eclipse ought to have been A. D. 1670. *April* the 8th 43¹/₂ past 5 a Clock in the Evening. And because the Earth was then a small Matter nearer its Aphelion, than the Moon its Apogon, the Time would be a little anticipated on that Account. But then, because this Eclipse was towards Evening, it would be much more retarded on that Account, than anticipated on the other; and the main Part of the

the Eclipse would happen after Sun-set, and be here invifible.

To this Time if we add another Cycle, the next correspondent Eclipse will in like Manner be difcovered:

	y	d	h	i	''
To A. D. 1670. <i>April</i>	—	00	08	5	43 15
Add ———		18	11	7	43 15
Sum 1688. <i>April</i>		00	19	13	26 30

So that the next correspondent total Eclipse ought to have been *April* 20th, 26¹/₂ after one in the Morning; and was therefore, to be fure, utterly invifible to us here. To this Time if we add another Cycle, we have the next correspondent total Eclipse thus:

	y	d	h	i	''
To A. D. 1688 <i>April</i>		00	19	13	26 30
Add ———		18	10	07	43 15
Sum 1706 <i>May</i>		00	00	21	09 45

So that the next correspondent total Eclipse was to have been A. D. 1706, 9¹/₂ past Nine a-Clock in the Morning; which is not much before the Time when it was observ'd here; and was no other than that famous Eclipse which was total at *Cadiz*, *Barcelona*, *Marseilles*, *Geneva*, *Bern*, and *Zurich*; and became very remarkable for the raising of the Siege of *Barcelona* during that total Darkness: Though I have been inform'd by feveral there present, that it came to both Armies wholly unexpected, till the great Darkness of the Sky forced them to attend to it. Now this affords us alfo a remarkable Instance of the near Approximation of our *Grand Cycle*, both

both as to Time and Place. For if instead of tracing this correspondent Eclipse through the distinct Cycles we had at once taken our *Grand Cycle* of 54 Years, and 33 Days; we had come immediately to this Eclipse: And by allowing the Anticipation of 50' had been within about 20' of the Calculation at *London*.

To this Time if we add another single Cycle, we shall have the next correspondent Eclipse thus :

	y	d	h	m	sec
To A. D. 1706. <i>May</i> —	00	00	21	09	45
Add —————	18	10	07	43	15
Sum 1724 <i>May</i> —	00	11	04	53	00

So that we ought hence to expect the total Eclipse next *May* 11, 53' past 4 in the Afternoon. And because both the Position of the Sun and Moon in their Ellipses, and the more considerable Alteration from the Time of the Day, which is here much farther in the Evening than the last was in the Morning; oblige us to suppose about 1^h Retardation, we hence justly expect that this Eclipse will be the nearest Total at *London* about 40' past Six in the Evening; as the exactest Calculations do determine.

(12.) If we would now trace a few Lunar Eclipses by this Cycle, we may do it according to the following Examples :

A. D. 1683, *Feb.* 11. about 59' past 10 a-Clock at Night, Mr. *Hamstead* observ'd the Middle of a great and total Eclipse of the Moon at *Greenwich*. Proceed therefore as is already directed to find the next correspondent Eclipse of the Moon thus :

To

	y	d	h	m	s
To A. D. 1681 February	00	11	10	59	00
Add	18	11	07	43	15
Sum 1699 Febr.	00	22	18	42	15

So that this first correspondent total Eclipse of the Moon ought to have been Feb. 23. 1699, 42' $\frac{1}{4}$ past Six a-Clock in the Morning; or in the day-time, and so must needs have been in great Part to us invifible.

	y	d	h	m	s
To A. D. 1699, Feb.	00	22	18	42	15
Add	18	10	07	43	15
Sum 1717 March	00	05	02	25	30

So that the next correspondent total Eclipse of the Moon ought to have been March 5, 1717, 25' $\frac{1}{4}$ after Two a-Clock in the Afternoon; which was in the day-time alfo; and fo must equally with the former have been here invifible.

	y	d	h	m	s
To A. D. 1717, March	00	05	02	25	30
Add	18	10	07	43	15
Sum 1736 March	00	15	10	08	45

So that the next total correspondent Eclipse of the Moon is hence to be expected A. D. 1736. March 15. 8 $\frac{1}{4}$ past 10 a-Clock at Night: which is about an Hour and half fooner than the Calculation. Which difference we fhall prefently find to be near the greateft Difference that can happen.

This

This may also be equally obtain'd by one entire *Grand Cycle* of 54 Years, and 33 Days; with the fore-mentioned Anticipation of 50' which from *Feb. 11. 1681* 10^h 59', brings us directly to *March 15, 1736*, 9' past 10 a-Clock at Night; or to somewhat above an Hour and half before the Calculation.

N. B. As to the proper Quantity of the several Alterations arising in each Period, which ought to be allowed for, they are nearly these: The Moon and Sun being about $31\frac{1}{3}$ in Diameter, and the Digits of their Observation being 12. while the Difference of the Moon's Latitude, as we have seen, is about $2' 36''$ or the Twelfth Part of those Diameters, it is plain that the mean natural Alteration of every Period in the same Circumstances is about one Digit; though less in the lesser, and greater in greater Latitudes: which in the Moon, whose Eclipse is to all Spectators the same, holds constantly: And though the diurnal Motion of the Earth removes all particular Places, so much each Period as to render this Rule less observable in Solar Eclipses, yet after each grand Period, which nearly restores their former Position, it will hold in a good Degree there also, I mean so as to alter about 3 Digits therein: But besides that of the Digits eclipsed, we ought also to see what Alteration in Time may happen to each Period. Now as to the Inequality of the Sun's Motion, it is as we have seen $10^{\circ} 8'$, and its greatest Velocity is at the Earth's Perihelion, and its least at its Aphelion: Its greatest Alteration therefore must be in *Aphelio* and *Perihelio*, and is the Difference of the Equation belonging every where to the Addition of $10^{\circ} 8'$, and is here $48''$, which Space the Earth goes

goes in about 20' of Time. So that the Difference of Time on this Account, must each Period be some Quantity less than 20'. And as to the Inequality of the Moon's Motion, it is also greatest at the Perigee and Apogee, and its greatest Alteration at the extreme Eccentricities of its Orbit is the Difference of the Equations at $2^{\circ} 51' \frac{1}{2}$ in Perigee and Apogee, according to those extreme Eccentricities : $= 9'$ which the Moon usually goes in somewhat less than 20'. So that the Difference of Time, on this Account, must each Cycle be some Quantity less than 20 Minutes also.

And now we come to the principal Alteration in Time that can happen in Eclipses ; though it belongs only to those of the Sun : And that is the Time of the Day when they happen in any particular Place. Now because the Center of the Moon usually goes over an entire Diameter of the Disk of the Earth, in about three Hours and an half, Part of which is almost always before, and Part after Noon : while the odd Hours of a small Cycle $7^h 43' \frac{1}{2}$, may reach equally from a Forenoon to an Afternoon's successive Eclipse ; 'tis possible, such an Eclipse may appear an Hour and three Quarters later than the Period it self would determine it. Tho' usually this Alteration will not be near so great ; especially when the Latitude of the Moon is very considerable. But then it is so easy to allow very nearly for this Inequality, upon a little Consideration, that it ought not to be objected against the Accuracy of this Period. If for Six Hours from Noon we allow about an Hour and half ; and for two Hours, three Quarters of an Hour ; we shall not err very much from the true Time.

Corollary. If we would know what is the greatest Inequality in Digits and Time in a grand Period, made up of Three common ones, or of 54 Years, 32 or 33 Days, besides the constant Anticipation of 50', we must say it may possibly, though it will very rarely, be almost thrice the Quantities already stated for a single Cycle: excepting the last and principal Difference, peculiar to Solar Eclipses: which is never much greater than that already mentioned.

N. B. If any are not contented to know these Matters by such Approximations, but desire the utmost Accuracy; they must either make use of Dr. *Halley's* Equations, fitted to this Cycle, when published; or rather make use of Mr. *Flamsteed's* or Dr. *Halley's* most accurate Astronomical Tables, when published; with that Trigonometrical Calculation afterward, which I publish and exemplify in this Paper. In the former Part of which Work, this Cycle, with its proper Equations, will, at least, save us the one half of our Calculation, if it will not bring us it self to that utmost Accuracy: which indeed is hardly to be expected from it.

So that, upon the whole, If we duly consider the particular Circumstances of the Sun and Moon, with those of the Aphelion, Apogee and Node, and with the Times of the Day or Night when the Cycle ends, and rightly apply them to this single Cycle and to this grand Cycle, we shall be able nearly to determine the correspondent Eclipses with very small Trouble or Calculation.

The

II.

The Plane in which the Center of the Moon moves in Eclipses, is not that of the Ecliptick, but of the Orbit of the Moon, consider'd with the Annual Motion : Or it is a Plane inclined to the Plane of the Ecliptick in an Angle of about $5^{\circ} 36'$. Which in the Calculation of Eclipses is usually stiled, *The Angle of the Moon's visible Way*. This principal Plane I call, *The Lunar Plane*.

III.

This *Lunar Plane* cuts the Sphere of the Earth, consider'd without its diurnal Motion, in a Circle whose Pole or Vertex is distant from the Pole of the Ecliptick in the same Angle. This Circle I call *The Lunar Circle*.

IV.

In Eclipses which happen at the Solstices, and in the Nodes of the Moon's Orbit, the Distance of these Poles is exactly *Eastward* or *Westward*. In those which happen at the Equinoxes and Nodes, the Distance is exactly *North* and *South*. But in all other Cases it is Oblique.

V.

The Angle of that Obliquity is always compos'd of the Distance of the Sun and Moon from the Solstitial Colure ; with the Distance of the same from the Nodes : And is sometimes the *Sum*, and sometimes the *Difference* of those Quantities.

VI.

The Distance between this Vertex and the Pole of the Earth, when Eclipses happen at the Solstices, is the *Sum* or *Difference* of the two fore-mentioned Angles of Inclination; the one, of the Pole of the Equator and of the Pole of the Ecliptick = $23^{\circ} 29'$; the other of the Pole of the Ecliptick, and of the Vertex of the Lunar Circle = $5^{\circ} 36'$ nearly. But in all other Cases a Spherical Triangle must be solv'd, in order to find that Distance : Of which hereafter.

VII.

Since the Solstitial Colure is a great Circle, that is also a Meridian, or passes through the Poles of the Earth and Ecliptick : And since besides the Distance between the Vertex of the Lunar Circle and the Pole of the Earth, we shall want the Angle included between the Colure and that Line; this also must be obtained by the like Solution of a Spherical Triangle : Of which hereafter.

VIII.

Since the great Circle that passes thro' the Poles of the Earth, and of the Lunar Circle, and that alone cuts both those Circles, and their Parallels at Right Angles, That Meridian, and that alone wherein that Distance lies, is perpendicular to the *Path of the Moon's Center* along the other; and will determine the Point in that Path wherein the Center of the Shadow cuts that Meridian at Right-Angles and approaches nearest of all to the Pole of the Earth : And indeed lays the Foundation of our future Calculations. This Meridian I call *The Primary Meridian*.

IX. The

IX.

The angular Distance about the Pole of the Earth, of the Meridian that is directed to the Sun at the middle Point of the whole Eclipse from the primary Meridian, is composed of the Sum or Difference of the Angle made by the Solstitial Colure and the primary Meridian; and of the Complement of the Sun's Right Ascension at the middle of the general Solar Eclipse. Which Angle is of the greatest Consequence in our future Calculations. This Angle I call *the Primary Angle*.

X.

Since the Motion of the Center of the Shadow of the Moon, in Solar Eclipses, is nearly even, and nearly recti-linear; since it is also in the Plane of the Lunar Circle; and is all one as if it were along a Line that touched that Circle at the Middle of the general Eclipse, the Point of Contact; we must divide each Quadram of 90 Degrees into 90 or 180 unequal Parts: but so that the Difference of the Sines of those unequal Angles may be equal, and $\frac{1}{90}$ or $\frac{1}{180}$ of the entire Radius: That so the first Sine $\frac{1}{90}$ may be $\frac{1}{100000}$; the second $\frac{2222}{100000}$; the third $\frac{3333}{100000}$, &c. and this from the Table of natural Sines, with their corresponding Arcs or Angles at the Vertex, as follows:

Arcs.

Arcs.	Diff. Sines Equal.
Diff. ° /	
19.1 0 18	$\frac{1}{8}$
19.1 0 38	1
19.1 0 57	$\frac{1}{8}$
19.1 1 16	2
19.1 1 35	$\frac{1}{8}$
19.1 1 54	3
19.1 2 13	$\frac{1}{8}$
19.2 2 33	4
19.2 2 52	$\frac{1}{8}$
19.2 3 11	5
19.2 3 30	$\frac{1}{8}$
19.2 3 49	6
19.2 4 8	$\frac{1}{8}$
19.2 4 28	7
19.3 4 47	$\frac{1}{8}$
19.3 5 6	8
19.3 5 25	$\frac{1}{8}$
19.3 5 44	9
19.3 6 3	$\frac{1}{8}$
19.4 6 23	10
19.4 6 42	$\frac{1}{8}$

Arcs.	Diff. Sines Equal.
Diff. ° /	
19.4 7 1	11
19.4 7 20	$\frac{1}{8}$
19.5 7 40	12
19.5 7 59	$\frac{1}{8}$
19.5 8 18	13
19.5 8 37	$\frac{1}{8}$
19.6 8 57	14
19.6 9 16	$\frac{1}{8}$
19.6 9 36	15
19.6 9 55	$\frac{1}{8}$
19.7 10 14	16
19.7 10 33	$\frac{1}{8}$
19.7 10 53	17
19.7 11 12	$\frac{1}{8}$
19.7 11 32	18
19.7 11 51	$\frac{1}{8}$
19.7 12 11	19
19.8 12 30	$\frac{1}{8}$
19.8 12 50	20
19.8 13 10	$\frac{1}{8}$
19.8 13 30	21

Arcs.

Arcs.	Diff. ° /	Diff. Sines Equal.
19.8		
13 49	$\frac{1}{2}$	
19.9		
14 9	.22	
20		
14 28	$\frac{1}{2}$	
20		
14 48	.23	
20		
15 8	$\frac{1}{2}$	
20		
15 28	.24	
20		
15 48	$\frac{1}{2}$	
20.1		
16 7	.25	
20.1		
16 27	$\frac{1}{2}$	
20.1		
16 47	.26	
20.2		
17 7	$\frac{1}{2}$	
20.2		
17 27	.27	
20.2		
17 47	$\frac{1}{2}$	
20.2		
18 8	.28	
20.3		
18 28	$\frac{1}{2}$	
20.3		
18 48	.29	
20.3		
19 8	$\frac{1}{2}$	
20.3		
19 28	.30	
20.4		
19 48	$\frac{1}{2}$	
20.4		
20 9	.31	
20.4		
20 29	$\frac{1}{2}$	

Arcs.	Diff. ° /	Diff. Sines Equal.
20.5		
20 50	.32	
20.5		
21 10	$\frac{1}{2}$	
20.5		
21 30	.33	
20.6		
21 50	$\frac{1}{2}$	
20.6		
22 11	.34	
20.7		
22 32	$\frac{1}{2}$	
20.7		
22 53	.35	
20.8		
23 13	$\frac{1}{2}$	
20.9		
23 34	.36	
21		
23 55	$\frac{1}{2}$	
21		
24 17	.37	
21.1		
24 38	$\frac{1}{2}$	
21.1		
24 59	.38	
21.2		
25 20	$\frac{1}{2}$	
21.2		
25 41	.39	
21.2		
26 2	$\frac{1}{2}$	
21.3		
26 23	.40	
21.3		
26 44	$\frac{1}{2}$	
21.4		
27 6	.41	
21.4		
27 27	$\frac{1}{2}$	
21.6		
27 49	.42	

Area.

Arcs.	Diff. ° /	Diff. Sines Equal.
21.7	28 11	$\frac{1}{2}$
21.8	28 32	43
21.9	28 54	$\frac{1}{2}$
22.0	29 16	44
22.1	29 38	$\frac{1}{2}$
22.2	30 0	45
22.3	30 22	$\frac{1}{2}$
22.4	30 44	46
22.6	31 6	$\frac{1}{2}$
22.8	31 51	47
23.1	32 29	$\frac{1}{2}$
23.2	32 14	48
23.4	32 36	$\frac{1}{2}$
23.5	32 59	49
23.6	33 22	$\frac{1}{2}$
23.6	33 45	50
23.7	34 9	$\frac{1}{2}$
23.8	34 31	51
23.9	34 54	$\frac{1}{2}$
23.9	35 18	52
24.1	35 41	$\frac{1}{2}$

Arcs.	Diff. ° /	Diff. Sines Equal.
24.2	36 4	53
24.3	36 27	$\frac{1}{2}$
24.4	36 52	54
24.6	37 16	$\frac{1}{2}$
24.7	37 40	55
24.8	38 4	$\frac{1}{2}$
24.9	38 29	56
25.1	38 54	$\frac{1}{2}$
25.3	39 18	57
25.4	39 43	$\frac{1}{2}$
25.5	40 7	58
25.7	40 32	$\frac{1}{2}$
25.8	40 57	59
25.9	41 22	$\frac{1}{2}$
26.1	41 48	60
26.3	42 14	$\frac{1}{2}$
26.4	42 40	61
26.5	43 6	$\frac{1}{2}$
26.7	43 32	62
26.9	43 58	$\frac{1}{2}$
27.3	44 25	63

Arcs.

Arcs.		Diff. Sines Equal.	Arcs.		Diff. Sines Equal.
Diff. °	'		Diff. °	'	
27.6			34		
	44.52	$\frac{1}{2}$		55.19	$\frac{1}{2}$
27.8			34.4		
	45.19	64		55.52	$\frac{1}{2}$
28.2			34.8		
	45.46	$\frac{1}{2}$		56.26	$\frac{1}{2}$
28.4			35.0		
	46.14	65		57.1	$\frac{1}{2}$
28.5			35.4		
	46.42	$\frac{1}{2}$		57.37	$\frac{1}{2}$
28.6			35.8		
	47.10	66		58.13	$\frac{1}{2}$
28.7			36.3		
	47.38	$\frac{1}{2}$		58.49	$\frac{1}{2}$
28.8			37		
	48.6	67		59.16	$\frac{1}{2}$
28.9			37.8		
	48.36	$\frac{1}{2}$		60.4	$\frac{1}{2}$
29			38.4		
	49.4	68		60.42	$\frac{1}{2}$
30			39.8		
	49.33	$\frac{1}{2}$		61.22	$\frac{1}{2}$
30			40.9		
	50.3	69		62.2	$\frac{1}{2}$
30.2			41.8		
	50.33	$\frac{1}{2}$		62.44	$\frac{1}{2}$
30.4			42.5		
	51.3	70		63.26	$\frac{1}{2}$
30.6			43		
	51.33	$\frac{1}{2}$		64.9	$\frac{1}{2}$
31			44.1		
	52.4	71		64.54	$\frac{1}{2}$
31.5			46		
	52.35	$\frac{1}{2}$		65.40	$\frac{1}{2}$
32			48		
	53.7	72		66.26	$\frac{1}{2}$
32.5			50		
	53.39	$\frac{1}{2}$		67.15	$\frac{1}{2}$
33			52		
	54.12	73		68.5	$\frac{1}{2}$
33.5			53		
	54.45	$\frac{1}{2}$		68.58	$\frac{1}{2}$

Arès.	Diff. Sines	Arès.	Diff. Sines
Diff. °	Equil.	Diff. °	Equal.
54		(86)	
69 52	$\frac{1}{2}$	77 54	88
56		(97)	
70 49	85	79 31	$\frac{1}{2}$
59		(116)	
71 48	$\frac{1}{2}$	81 27	89
63		(69)	
72 51	86	82 36	$\frac{1}{4}$
67		(81)	
73 58	$\frac{1}{2}$	83 57	$\frac{1}{2}$
72		(105)	
75 16	87	85 42	$\frac{1}{2}$
78		(258)	
76 28	$\frac{1}{2}$	90 00	90

Where the Angle made by the Lunar Circle, and the Parallel of the Latitude is considerable ; instead of the first Number 19^L1 you must take for the Numbers following ; being in a reciprocal Proportion to the Secants of those Angles.

Angl.	
0	19 ^L 1
5	19 ^L 1
10	18 ^L 8
15	18 ^L 4
20	17 ^L 9
25	17 ^L 3
30	16 ^L 5

N. B. Where the Parallel is different from that at the very Middle of the Eclipse; as it usually is; you must increase or decrease the same Numbers, 19.1 , &c. in the Proportion of the Sines of the Latitude thus:

00	000
10	174
20	342
30	500
40	643
50	766
60	867
70	940
80	985
90	1000

E. G. If the Co-latitude at the Middle of the Eclipse be 60° , and come to be 40° ; say, As 867 to 643, So is 19.1 to 14.2 , which is there to be taken in its stead.

N. B. The Hint that I had several Years ago, that in the Determination of Solar Eclipses the Equality of the Difference of Sines was made use of by Dr. Halley, was the Occasion of the Discoveries in these Papers.

XI.

The perpendicular Distance of every Point of the Penumbra ; and the like Distance of every Point of the total Shadow from the Path of the Moon's Center , may be discovered by Tables made from the natural Sines ; where those Sines themselves, as before, differ equally, or in arithmetical Progression ; according to the Duration of the whole Eclipse , or of total Darkness : and their Co-sines correspond to the Distances from that Path. Both which Tables here follow :

For

*For entire Eclipses : Which are here supposed
108' $\frac{1}{2}$ long in the Middle, and the Semi-
diameter of the Penumbra 2000 Miles.*

Duration in Minut.	Distance in Miles.	Duration in Minut.	Distance in Miles.	Dig ^{ts}
1	2000	37	1879	
2	1999	38	1872	
3	1999	39	1866	
4	1999	40	1859	
5	1998	41	1852	
6	1997	42	1844	
7	1996		1838	1
8	1995	43	1835	
9	1993	44	1828	
10	1991	55	1820	
11	1990	56	1812	
12	1988	47	1803	
13	1986	48	1795	
14	1984	49	1786	
15	1981	50	1776	
16	1979	$\frac{1}{2}$	1771	
17	1976	51	1766	
18	1973	$\frac{1}{2}$	1761	
19	1970	52	1756	
20	1966	$\frac{3}{4}$	1751	
21	1963	53	1746	
22	1959	$\frac{1}{2}$	1741	
23	1955	54	1736	
24	1951	$\frac{1}{2}$	1731	
25	1946	55	1725	
26	1942	$\frac{1}{2}$	1720	
27	1937	56	1714	
28	1932	$\frac{1}{2}$	1708	
29	1927	57	1703	
30	1922	$\frac{1}{2}$	1697	
31	1917	58	1691	
32	1911	$\frac{1}{2}$	1685	
33	1905	59	1679	
34	1899		1676	2
35	1893	$\frac{1}{2}$	1672	
36	1886	60	1667	

Dura-

Duration in Minut.	Distance in Miles.	Duration in Minut.	Distance in Miles.	Distance
$\frac{1}{2}$	1662	80	1354	4
61	1655		1350	
$\frac{1}{2}$	1648	$\frac{1}{2}$	1344	
62	1642	$\frac{1}{2}$	1338	
$\frac{1}{2}$	1635	$\frac{1}{2}$	1322	5
63	1628	$\frac{1}{2}$	1312	
$\frac{1}{2}$	1621	$\frac{1}{2}$	1302	
64	1614	$\frac{1}{2}$	1294	
$\frac{1}{2}$	1609	$\frac{1}{2}$	1280	
65	1603	84	1269	
$\frac{1}{2}$	1596	$\frac{1}{2}$	1258	
66	1589	$\frac{1}{2}$	1246	
$\frac{1}{2}$	1582	$\frac{1}{2}$	1234	
67	1575	86	1222	
$\frac{1}{2}$	1568	$\frac{1}{2}$	1110	
68	1561	87	1098	
$\frac{1}{2}$	1554		1188	6
69	1546	$\frac{1}{2}$	1186	
$\frac{1}{2}$	1538	$\frac{1}{2}$	1178	
70	1534	$\frac{1}{2}$	1160	
$\frac{1}{2}$	1528	89	1147	
71	1521	$\frac{1}{2}$	1134	
$\frac{1}{2}$	1514	90	1121	
	1513	$\frac{1}{2}$	1107	
72	1506	91	1098	
$\frac{1}{2}$	1498	$\frac{1}{2}$	1079	
73	1490	92	1065	
$\frac{1}{2}$	1482	$\frac{1}{2}$	1050	
74	1474	93	1034	
$\frac{1}{2}$	1456		1025	6
75	1448	$\frac{1}{2}$	1017	
$\frac{1}{2}$	1439	94	1000	
76	1430	$\frac{1}{2}$	983	
$\frac{1}{2}$	1424	95	971	
77	1412	$\frac{1}{2}$	955	
$\frac{1}{2}$	1403	96	937	
78	1394	$\frac{1}{2}$	919	
$\frac{1}{2}$	1384	97	902	
79	1374	$\frac{1}{2}$	884	
$\frac{1}{2}$	1364	98	864	

Duration

Duration in Minut.	Distance in Miles.	Duration in Minut.	Distance in Miles.	Distance
	863	7	550	537
89 $\frac{1}{2}$	844	$\frac{3}{4}$	534	9
99 $\frac{1}{2}$	824	105	517	
100 $\frac{1}{4}$	804	$\frac{1}{4}$	499	
$\frac{1}{4}$	784	$\frac{1}{2}$	482	
$\frac{1}{2}$	773	$\frac{3}{4}$	463	
$\frac{3}{4}$	762	106	443	
101 $\frac{1}{4}$	751	$\frac{1}{4}$	421	
$\frac{1}{2}$	739	$\frac{1}{2}$	399	
$\frac{3}{4}$	728	$\frac{3}{4}$	376	
102 $\frac{1}{4}$	716		375	10
$\frac{1}{2}$	704	107	352	
$\frac{3}{4}$	691	$\frac{1}{4}$	320	
103 $\frac{1}{4}$	679	$\frac{1}{2}$	288	
$\frac{1}{2}$	666	$\frac{3}{4}$	258	
$\frac{3}{4}$	653	108	226	
104 $\frac{1}{4}$	639		180	212
$\frac{1}{2}$	625	$\frac{1}{4}$	119	
$\frac{3}{4}$	611	$\frac{1}{2}$		50
105 $\frac{1}{4}$	597		000	12
$\frac{1}{2}$	582			
$\frac{3}{4}$	566			

For

For Total Darkness of 166"

Duration in Sec ^{ds} .	Distance in Miles.	Duration in Sec ^{ds} .	Distance in Miles.
"	"	"	"
1	50.0	37	49.0
2	50.0	38	49.0
3	50.0	39	49.0
4	50.0	40	48.9
5	50.0	41	48.9
6	50.0	42	48.8
7	50.0	43	48.7
8	50.0	44	48.6
9	50.0	45	48.5
10	50.0	46	48.4
11	49.9	47	48.2
12	49.8	48	48.1
13	49.8	49	47.9
14	49.8	50	47.7
15	49.7	51	47.6
16	49.7	52	47.5
17	49.7	53	47.4
18	49.6	54	47.3
19	49.6	55	47.2
20	49.6	56	47.1
21	49.6	57	47.0
22	49.5	58	46.9
23	49.5	59	46.8
24	49.5	60	46.6
25	49.4	61	46.5
26	49.4	62	46.4
27	49.4	63	46.3
28	49.3	64	46.2
29	49.3	65	46.0
30	49.3	66	45.9
31	49.2	67	45.8
32	49.2	68	45.6
33	49.2	69	45.5
34	49.1	70	45.3
35	49.1	71	45.2
36	49.1	72	45.1

Dura-

Duration in Sec ^{ds} .	Distance in Miles.	Duration in Sec ^{ds} .	Distance in Miles.
73	45.0	111	37.4
74	44.8	112	37.1
75	44.6	113	36.8
76	44.5	114	36.5
77	44.3	115	36.2
78	44.1	116	35.9
79	44.0	117	35.6
80	43.8	118	35.3
81	43.7	119	35.0
82	43.5	120	34.7
83	43.3	121	34.4
84	43.2	122	34.1
85	43.0	123	33.8
86	42.9	124	33.4
87	42.7	125	33.0
88	42.5	126	32.7
89	42.3	127	32.4
90	42.1	128	32.1
91	41.9	129	31.7
92	41.7	130	31.3
93	41.5	131	30.9
94	41.3	132	30.5
95	41.1	133	30.1
96	40.9	134	29.7
97	40.7	135	29.3
98	40.5	136	28.9
99	40.3	137	28.5
100	40.0	138	28.0
101	39.8	139	27.5
102	39.6	140	27.1
103	39.4	141	26.6
104	39.1	142	26.1
105	38.8	143	25.6
106	38.6	144	25.1
107	38.4	145	24.6
108	38.1	146	24.1
109	37.9	147	23.5
110	37.6	148	23.0

F

De-

Duration in Sec ^{ds} .	Distance in Miles.	Duration in Sec ^{ds} .	Distance in Miles.
//		//	
149	22.4	159	14.9
150	21.8	160	14.0
151	21.1	161	13.9
152	20.5	162	13.7
153	19.8	163	13.4
154	19.1	164	8.9
155	18.3	165	7.0
156	17.6	166	4.4
157	16.7	166 $\frac{2}{3}$	0.0
158	15.9		

Semidiameter of the Umbra or Total Dark-
ness = 50 Miles.

XII.

The Angles at the Vertex of the Lunar Circle, on each Side of the Point of Contact, by Reason of the perpendicular Situation of that Axis to its own Circle ; are always right Angles : Only diminish'd in the Proportion of the Minutes describ'd by the Annual Motion during the Continuance of the Eclipse. Thus in our present Eclipse, which retains the Center of the Shadow near three Hours upon the Earth's Disk, in which Time the annual Motion amounts to about 8' ; each of those right Angles in Strictness are to be esteem'd only $89^{\circ} 56'$, and both together $179^{\circ} 52'$. Only because the Refraction of the Rays of the Sun through our Atmosphere, requires a somewhat greater Increase of this Angle, than the annual Motion requires its Diminution, I shall wholly omit it, in all my Calculations hereafter.

XIII.

XIII.

The Angles made at the Poles of the Earth, which shew the Difference of the two extreme Meridians, and limit the Extent of the entire central Eclipse, by reason of the Obliquity of the Earth's Axis to that Lunar Circle, are usually unequal to one another; and more or fewer than twice 90° , as the Eclipse happens at different Latitudes of the Moon, and Times of the Year.

XIV.

The Meridian that passes through the Middle or Central Point of general Solar Eclipses, is the same with that which passes through the Center of the Sun at the same Time, when the Moon has no Latitude, and the Eclipses are Central in the Plane of the Ecliptick; as also when they happen in either Solstices. Otherwise the Moon's Latitude being taken perpendicular to the Plane of the Ecliptick; and the nearest Distance of the Moon's Motion being taken perpendicular to the Lunar Circle, while the Meridians always pass through the Poles of the Earth; these Two Meridians will, generally speaking, be different, and their included Angle no otherwise to be known than by Trigonometry, as will appear hereafter.

XV.

The Dimensions of the Pénumbra, or entire Eclipse, and the Extent of the total Shadow on the Earth, are continually different, according to the different Elevations of the Sun and Moon above any particular Horizon. For as the Moon is about the same Distance from every Place, when it is in its Horizon, as it is from the Earth's Cen-

ter it self; with regard to which Center alone our first Calculations are always made: So when it is in the Zenith of any Place, it is one Semi-diameter of the Earth nearer it; which Semi-diameter being usually $\frac{1}{3}$, and at our next Eclipse $\frac{1}{2}$ of its entire Distance, as will appear hereafter, will deserve an Allowance. Nor will any lesser Elevation of the Moon be wholly inconsiderable in Eclipses, but in all accurate Determinations thereof must be particularly computed, in order to the distinct Knowledge of the Extent of such Eclipses; especially of the Breadth of the Total Shadow therein. Accordingly we are to observe that this Breadth of the Total Shadow will certainly be at this Eclipse considerably greater over *North America*; where the Luminaries are greater elevated above the Horizon; than over *Europe*, where they are much nearer it; as this Calculation requires: Of which hereafter.

XVI.

The *Figure* of the entire *Penumbra*, or general Eclipse; and of the *Umbra*, or Total Darkeness; as they appear upon every Country, is different, on account of the different Obliquity of every Horizon; and will make Ovals or Ellipses of different Species perpetually. This in the vast *Penumbra* is best understood by such an Instrument as my *Copernicus*; or by the Perusal of a very scarce Book written by P. Courcier, (*Philos. Transact.* N^o. 343. p. 259.) and cited by Dr. Halley: Which distinctly treats of the Intersection of a Conical and Spherical Surface. But in the smaller *Umbra*, or Total Darkeness, which is confined to a much narrower Compass, it very nearly approaches to the Intersection of a conick Surface with a Plane, which is a true Ellipsis.

XVII.

XVII.

The *Species* of that Ellipsis depends on the Sun's Altitude above the Horizon at the Time of Total Darkness ; as does the *Position* of its longer Axis on the Azimuth of the Sun at the same Time. Nor is it at all necessary that the Direction of this or any other Ellipsis should be along either of the Axes, but may as well be along any other Diameter whatsoever.

XVIII.

The *Direction* of the Center of the Shadow is according to the Direction of the Moon's Motion, along the Plane of the Lunar Circle ; as compounded with the diurnal Motion, or with the Direction and Velocity of those Parts of the Earth over which it passes ; and will be hereafter brought to Calculation. And indeed this Angle may be had, either by finding the several Points of the Path of the Moon's Way upon the Earth, in as many Meridians as we please, and drawing a curve Line through those Points ; or by solving a Spherical Triangle, whose Sides are the Complements of the Latitudes of two neighbouring Places equally distant, *East* and *West*, from the Place you work for ; and whose included Angle is the Angle at the Pole suited to the Difference of their Meridians ; and taking half the the two Angles at the Base, the one internal and the other external, for the Angle desir'd : Of which hereafter.

XIX.

Every Ellipsis, made by the oblique Section of a Cone, has the Intersection of the (*Fig. 1.*) Axis of the Cone *C* at some Distance from the Center of
the

the Ellipsis D . And the Proportion of those unequal Divisions BC and CA are the same with that of the Sides of the Cone VB and VA : As appears by the (*Elem.* III. 6.) Elements of *Euclid*. Whence it is evident, that the proper Center of the total Shadow in Eclipses of the Sun, or that made by the Axis of the Cone, is not the same with the Center of the Elliptick Shadow; and that the Proportion of its Distance from that Center may be easily determin'd by the Proposition here refer'd to: Of which more hereafter.

Scholium. This Ellipsis, when the Sun is of a considerable Altitude, is almost an exact one; but when the Sun is near the Horizon, it will be very long, and so less exact; because the Spherical Surface of the Earth is at that Distance more remote from a Plane.

XX.

The perpendicular *Breadth* of the Shadow is neither that of the longer, nor that of the shorter Axis: But that of the two longest Perpendiculars (*Fig. 2.*) AB and CD drawn from the Tangents parallel to the Diameter DB , along which the Direction of the Motion is: The length of which Perpendiculars will be hereafter determined.

XXI.

The *Velocity* of the Motion of the Center of the Shadow is unequal; not only on account of the Difference of the Moon's own Motion, at the beginning and ending of the entire Eclipse; which indeed is very inconsiderable; but chiefly by reason of the Difference of the Obliquity of
the

the Horizon all the Way of its Passage. However, since the several Points may, in all Meridians be distinctly found by Trigonometry, as we shall shew presently, this Inequality need create us no new Difficulty in the Determination of Eclipses.

XXII.

The Number of *Digits* eclips'd, which are twelfth Parts of the Sun's Diameter, with Sexagesimal Parts of the same Digits, are always to be estimated as distinct from the total Shadow; and may be discovered by the help of the foregoing Table, p. 29, 30, 31. Where the Digits are already noted at every proper Distance from the Path of the Moons Center; and where the intermediate Fraction $\frac{1}{702.5}$ is more exact than $\frac{1}{700}$; but which, by dividing that Number $\frac{1}{700}$ by 2.7, will give those Sexagesimals, without any farther Trouble. The Application of that Table will be taught hereafter.

XXIII.

The *Distance* of the Vertex of the conical Shadow of the Moon, which sometimes just reaches the Surface of the Earth, as in total Eclipses *sine morâ*, Sometimes does not reach it; as in Annular Eclipses; and sometimes would overreach it, if it were not intercepted; as in total Eclipses *cum morâ*; may be easily discovered at any time by the Analogy following: As PC the (Fig. 3.) Semidiameter of the Moon $\equiv 941$ geographical Miles is to CV the Distance of the Moon from that Vertex $\equiv 215000 ::$ So is $Rr \equiv 45$, the smallest Semidiameter of the total Shadow, which is the same as of the circular Shadow itself, to sV , the Distance of that Vertex therefrom

from = 10316, which is thus : 941 : 215000 :: 48
: 10316.

XXIV.

The Determination of the Circumstances of Solar Eclipses, for any given Distance from the Path of the Moon's Center, either way, has no new Difficulty in it; but is to be made just as is that for the Center of the Penumbra. Only the Quantity of the Distance of the Vertex of the Lunar Plane from that Circle will be different; as the Path of the Moon's Center it self might be at another Eclipse, of otherwise the same Circumstances.

XXV.

If Two Bodies *A* and *B* set out together, the one from *A*, the other from *B*: and move evenly forward in a known Proportion as to Velocity; the Point *C* will be determined where the swifter will overtake the slower, and they will be coincident. Thus if the Velocity of *A*, (*Fig. 4.*) be to that of *B*, as 5 to 1, the Proportion of the Lines *AB* to *BC* will be as 4 to 1, and if we add 1 to 4 = 5 we have the place *C* where the swifter will overtake the slower. Thus if their Velocities be to each other as 5.48 to 1, the Lines of their Motion *AB* and *BC*, will be as 4.48 to 1. So that if we take in the former Case $\frac{1}{5}$ of *AB* and in the other $\frac{1}{5.48}$ of *AB*, and add it to *AB*, we gain *AC* the Distance of the Point *C* from *A*.

Corollary. If therefore *A* represent the Center of the Shadow of a Solar Eclipse, as it is plac'd at the Middle of the general Eclipse; and *B Greenwich* at the same Moment of absolute Time; and at a known Distance from the middle Point;

Point ; and if the Velocity of the Center of the Shadow along the Circle be to the Velocity of *Greenwich* in its diurnal Motion as 5₁·48 to 1, if we ad 4₁·48 of their Distance at their setting out to that known Distance, we obtain the Point or Place where the Center of the Eclipse will overtake *Greenwich*, or the Time when the Eclipse will be at the Meridian of *Greenwich*. And this whether the Center and *Greenwich* move along the same Line as *ABC*, or two different Lines, as *ABC* and *abc*.

XXVI.

The Duration of Solar Eclipses is different, according as their Middle happens about Six in the Morning or Evening ; or about Noon ; or about any intermediate Time. If that happens about Six a-Clock, Morning or Evening, the diurnal Motion then neither much conspires with, nor opposes the proper Motion of the Center of the Shadow ; and the Duration is almost the same as it would be if the Earth had no diurnal Motion at all. If that happens about Noon, the diurnal Motion most of all conspires with that proper Motion of the Center , and makes the Duration of the Eclipse the longest possible. If it happens in the intermediate Times, the diurnal Motion, in a less Degree, conspires with the other Motion, and makes the Duration of a mean Quantity, between that of the other Cases. But if it happens considerably before Six a-Clock in the Morning, or after Six a-Clock in the Evening, the diurnal Motion is backward, and shortens that Duration proportionably. Of the Quantity of which Duration we shall enquire more hereafter.

P R O B L E M S.

I.

To find the nearest Distance of the Path of the Moon's Center, to the Center of the Disk of the Earth, as seen at the Distance of the Moon in the total Eclipse of the Sun A. D. 1724. May 11^o. P. M.

This is equal to the Moon's true Latitude at the Time of the Conjunction in her own Orbit; and is set down in the Calculation 32' 19".

II.

To find the Sun's *Declination* at the Middle of the Eclipse.

As the Radius of the Circle: is to the Sine of the Sun's Longitude at that Time = 61' 39" :: So is the Sine of the Sun's greatest Declination, = 23° 29' : to the Sine of the Sun's Declination then.

Rad.	10.	000000	
S. 61° 39'	9.	944514	
S. 23° 29'	9.	600409	• "
S.	8.	544923	= 20 32
			= Declination ☉.

III.

To find the Sun's *Right Ascension* for the same Time.

As the Radius : to the Cofine of the Sun's greatest Declination :: So is the Tangent of the Sun's Latitude: to the Tangent of the Sun's right Ascension. Rad.

Rad.	10.
Sin. 66 31	9. 962453
Tang. 61 39	10. 267952
Tang. Right Ascen. 10. 230405	= 59 31
	Compl. 30 29

IV.

To find the *Distance* between the Vertex of the Lunar Circle and the Pole of the Earth.

Let (Fig. 5.) *EP* represent the Distance between the two Poles of the Earth, and of the Ecliptick : = $23^{\circ} 29'$. *EV* the Distance between the Pole of the Ecliptick, and the Vertex of the Lunar Circle : = $5^{\circ} 36'$. *PEV* the Angle made by the Solstitial Colure *PE* : (in which the two Poles of the Earth, and of the Ecliptick always are) and that Arc *EV*. And because the Sun is here $28^{\circ} 21'$ distant from that Colure, which is the Complement of its Longitude from *Aries* ; and the Ascending Node, or Argument of Latitude is then $5^{\circ} 49'$ distant from the Sun backward ; The Sum of these Numbers gives $34^{\circ} 10'$, whose Complement is the Angle *VER* = $55^{\circ} 50'$. In order then to gain *VP* proceed thus :

R.	10.
CS <i>VER</i> . 55 50.	9. 749429
T. <i>VE</i> . 5 36	8. 991451
T. <i>RE</i>	18. 740880 = 3. 10.
	+ 23. 29.
	= 26. 39.

Then say,

CS. <i>RE</i>	3 9	9. 992343
CS. <i>VE</i>	5 36	9. 997922
CS. <i>PR</i>	26 39	9. 951222
		19. 949144
CS. <i>PV</i>	= 9. 942801	= 27. 0
	G 2	V. In

V.

In the same Triangle VPE , to find the Angle VPE , included between the Colure EP , and the Prime Meridian PV .

$$\begin{array}{rcl}
 \text{S. } PV & 27 & 0. 9. 657047 \\
 \text{S. } VER & 55 & 50. 9. 917719 \\
 \text{S. } VE & 5 & 36. 8. 989374 \\
 & & 18. 507098 \\
 \text{S } VPE & = & 9. 249961 = 10 \ 15
 \end{array}$$

Corollary, The *Primary Angle*, composed of the Complement of the Right Ascension, and of this Angle VPE , is $= 40^\circ 44'$.

$$\begin{array}{r}
 \text{Compl. Rad. } 30^\circ 29' \\
 + 10 \ 15 \\
 = 40 \ 44
 \end{array}$$

VI.

To find the *Distance* of the Pole or Vertex of the Lunar Circle from the Circle it self.

As the Semidiameter of the Earth's Disk $= 61' 28''$: To the Latitude of the Moon, or nearest Distance to the Path of the Moon's Center from the Center of the Disk; $32^\circ 19'$:: So is the Radius: to the Sine of the Complement of that Distance. In Decimals thus: $61' 63 : 32' 32 :: 10000 : 5244 = \text{S. } 31^\circ 38'$, whose Complement is $58^\circ 22'$: equal to the Distance of the Lunar Circle from its Vertex.

VII.

To find the *Angle* included between the Meridian that passes through the Center of the general Eclipse, and that passing through the Center of the Sun at the same Time.

In

In the Triangle (*Fig. 6.*) MSP where the Side SP is already given, = Complement of the Sun's Declination; find the Angle MSP thus:

R.	10.
S. of the Sun's Distance	
from the Solstitial Co-	
lure :	= 28° 21' 9. 676562
S. of the Sun's greatest	
Declination	= 23° 29' 9. 600409
S. of an Angle	9. 276971 = 10. 54
To which add the Angle of the	
Moon's Way :	+ 5 36
The Sum is the Angle MSP	= 16. 30

Then, in the same Triangle $PM S$, we have two Sides : MS equal to the Latitude of the Moon, or the length of the Perpendicular to the Moon's way = 31° 38'. and PS = 69° 28'. and the included Angle MSP = 16° 30'. to find MPS thus:

R.	10.
CS. MSP .	16. 30. 9. 981774
T. SM .	31. 38. 9. 789585
T. RM .	= 9. 771359 = 30 34 MR
	+ 31 38 MS
Then say,	= 62 12 RS
S. SP .	69. 28. 9. 971493
S. SR .	62. 12. 9. 946738
T. MSP .	16. 30. 9. 471605
	9. 418343
T. MPS	= 9. 446850 = 15 38
	+ 40 44
	= 56 22 = An-
gle with the Primary Meridian.	

VIII.

To find the *Longitude* and *Latitude* of the Center of the Shadow at the Middle of the general Eclipse; or to solve the primary Triangle.

Under *Problem IV.* we have found the Distance of the Vertex of the Lunar Circle from the Pole of the Earth = $27^{\circ}. 0'$. Under *Problem VI.* we have found the Distance of that Vertex from its Circle = $58^{\circ}. 22'$; and under the last *Problem* we have found the Angle at the Pole of the Earth, between the Primary Meridian, and that Meridian which passes through the Center of the Eclipse, = $56^{\circ}. 22'$. From which *data* the Primary Triangle (*Fig. 7.*) is thus to be solv'd :

R.	10.
CS. $CP \angle$ 56. 22.	9. 743412
T. VP . 27. 0.	9. 707166
T. $P R$. = 15. 46.	9. 450578

Then say,

CS. VP 27. 0.	9. 949880
CS. PR 15. 46.	9. 983345
CS. VC 58. 22.	9. 719730
	19. 703075
CS. PC .	9. 753195 = 55. 30
	Deduct PR = 15. 46
	Rem ^t . = PC . = 39. 44
	Ergo Lat. = 50. 16

To find the Angle at the Vertex $CV P$, proceed in this Manner :

S. CV .

$$\begin{array}{rcl}
 \text{S. CV. } 58. 22. & 9. 930145 & \\
 \text{S. CPQ. } 56. 22. & 9. 920436 & \\
 \text{S. PC. } 39. 44. & 9. 805647 & \\
 & 19. 726083 & \\
 \text{S. CVP.} & = 9. 795938 = 38 \text{ } 41 &
 \end{array}$$

The following Analogy will give *VCP* the Complement of the Angle which the Direction of the Center of the Eclipse makes with the Meridian, that Direction being perpendicular to *VC*

$$\begin{array}{rcl}
 \text{S. CV. } 58. 22. & 9. 930145 & \\
 \text{S. CPQ. } 56. 22. & 9. 920436 & \\
 \text{S. VP. } 27. 0. & 9. 657047 & \\
 & 19. 577483 & \\
 \text{S. VCP.} & = 9. 647338 = 26. 21 & \\
 \text{Compl.} & = 63. 39 &
 \end{array}$$

Corollary. Hence we also learn the most *Northern Latitude*, where the Center of the Shadow will cross the Meridian at Noon, and at right Angles: And this without any particular distinct Calculation. For $VQ = 58^\circ 22' - VP 27^\circ 0' = PQ = 31^\circ 22'$. whose Complement = $58^\circ 38'$. is that very *Northern Latitude*.

IX.

To find the *Longitude* and *Latitude* of the Center of the Shadow, when it crosses the Meridian that passes through the Center of the Sun at the Middle of the Eclipse; or to solve the second principal Triangle,

If

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If in the foregoing Triangle we suppose the Angle at the Pole to be equal to the Primary Angle, or $40^{\circ} 44'$. we may thus solve this Triangle:

R. 10.

$$\begin{array}{rcl} \text{CS. CP} \angle 40. 44. & 9. & 879529 \\ \text{T. VP. 27. 0.} & 9. & 707166 \\ \text{T. PR.} & = & 9. 586695 = 21. 7 \end{array}$$

Then say,

$$\begin{array}{rcl} \text{CS. VP. 27. 0.} & 9. & 949880 \\ \text{CS. PR. 21. 7.} & 9. & 969811 \\ \text{CS. VC. 58. 22.} & 9. & 719730 \\ & 19. & 689541 \\ \text{CS. CR.} & = & 9. 739661 = 56. 42 \\ & & \text{Deduct } 21. 7 \\ & & \text{Remt. CP. } 35. 35 \\ & & \text{Ergo, Lat.} = 54. 25 \end{array}$$

To find the Angle at the Center of the Eclipse VCP, proceed thus:

$$\begin{array}{rcl} \text{S. VC. 58. 22.} & 9. & 930145 \\ \text{S. CP} \angle 40. 44. & 9. & 814607 \\ \text{S. VP. 27. 0.} & 9. & 657047 \\ & 19. & 471654 \\ \text{S. CVP.} & = & 9. 541509 = 20. 22 \end{array}$$

X.

To find the Longitude and Latitude of the Center of the Shadow at its Entrance on the Disk of the Earth: Or to solve the third Principal Triangle.

Add the vertical Angle already found $= 38^{\circ} 41'$ to a right Angle at the Vertex; $= 90 + 38^{\circ} 41' = 128^{\circ} 41'$ this is equal to the Angle at the

tex *CVP*. Subtract this Angle from two right Angles. $180^\circ - 128^\circ.741' = 51^\circ.19'$, in order to gain the Supplement, whose Sines, &c. are the same with the others. (*Fig. 8.*) Then say,

R. 10.

CS. *CVP*. 51. 19. 9. 795891

T. *VP*. 27. 00. 9. 707166

T. *VR*. = 19. 503057 = 17 40 = *VR*
+ 58 22 = *VC*
= 76 02 = *CR*

Then say,

CS. *VR*. 17. 40. 9. 979019

CS. *VP*. 27. 00. 9. 949880

CS. *CR*. 76. 02. 9. 382661

19. 332541

CS. *CP*. = 9. 353522 = 76: 57
Ergo, Lat. 13. 03

In order to find the Angle at the Pole *VP C*, whose Supplement is the Longitude of that Point where the Center of the Shadow enters the Disk of the Earth from the Primary Meridian, proceed thus :

S. *CP*. 76. 57. 9. 988636

S. *CVP*. 51. 19. 9. 892435

S. *CV*. 58. 22. 9. 930145

19. 822580

S. *VPC*. = 9. 833944 = 43 01

Suppl. 136 : 59

To find the Angle *VCP*, proceed thus :

H

S. *CP*.

$$\begin{array}{rcl}
 \text{S. CP. } 76. 57. & 9. & 988636 \\
 \text{S. CVP. } 51. 19. & 9. & 892435 \\
 \text{S. VP. } 27. 00. & 9. & 657047 \\
 & 19. & 549482 \\
 \text{S. VCP.} & = 9. & 560846 = 21. 20
 \end{array}$$

XI.

To find the Longitude and Latitude of the Center of the Shadow at its Exit from the Disk of the Earth ; or to solve the fourth Principal Triangle.

Subtract the Angle already found $38^\circ 41'$ from a right Angle $90^\circ - 38^\circ 41' = 51^\circ 19' =$ Angle at the Vertex PVC (Fig. 9.) Then say,

$$\text{R.} \quad 10.$$

$$\begin{array}{rcl}
 \text{CS. PVC. } 51. 19. & 9. & 795891 \\
 \text{T. PV. } 27. 00. & 9. & 707166 \\
 \text{T. VR. } 19. 503057 & = & 17 \ 40 \\
 & & \text{from } 58 \ 22
 \end{array}$$

$$\text{Then say,} \quad \text{Remnt } 40 \ 42 = \text{RC.}$$

$$\begin{array}{rcl}
 \text{CS. VR. } 17. 40. & 9. & 979019 \\
 \text{CS. PV. } 27. 00. & 9. & 949880 \\
 \text{CS. RC. } 40. 42. & 9. & 879746 \\
 & 19. & 829626 \\
 \text{CS. PC} & = 9. & 850607 = 44. 51. \\
 & & \text{Ergo, Lat. } 45. \ 9.
 \end{array}$$

In order to find the Angle $\angle PC$, or the Longitude of that Point where the Center of the Shadow departs out of the Disk of the Earth, from the Primary Meridian, proceed thus:

CP.

[51]

S. CP. 44. 51. 9. 848345
 S. PVC. 51. 19. 9. 892435
 S. VC. 58. 22. 9. 930145
 19. 822580

S. $\angle PC = 9. 974235 = 70. 27$
 $+ 136 59$
 $= 207 26$

And for the Angle VCP thus;

S. CP. 44. 51. 9. 848345
 S. PVC. 51. 19. 9. 892435
 S. VP. 27. 00. 9. 657047
 19. 549482

S. $VCP. = 9. 701137 = 30. 10.$

Corollary. Hence the Angular Motion of the Center of the Eclipse about the Pole of the Earth, if there were no diurnal Motion, is $207^{\circ}.26'$.

XII.

To find the Time in which the Center of the Shadow will go over the Diameter of the Lunar Circle.

Say, first, $35' 3 : 60' :: 123.26 : 209.6$; i.e. As the Number of Minutes of a Degree pass'd over in an Hour : to an Hour :: So is the entire Diameter of the Disk from the Calculation : to the Number of Minutes for that Passage.

Then say, $10000 : 8514 :: 209.6 : 178.4$ i.e. As the Radius : to the Sine of $58^{\circ} : 22' =$ the Distance of the Lunar Circle from its Pole or Vertex :: So are the Minutes of the Passage over the entire Diameter : to the Minutes of the Passage over this Chord $= 178' 24''$.

XIII.

To find the Proportion of the Velocities of the Center of the Shadow and of the diurnal Motion of the corresponding Point of the Earth at the Time of the Eclipse :

Say thus ; As $178^{\circ} 4'$ to $829^{\circ} 6' = 207^{\circ} 26'$, or as 1 to $4^{\circ} 64'$, so is the Time of the Center of the Eclipse's Motion over the Diameter of the Lunar Circle : to the Time of the diurnal Motion's going from the entrance to the Exit of the Center.

Corollary. Hence the real Angular Motion of the Center of the Eclipse about the Pole of the Earth, is no more than $162^{\circ} 40'$. For $4^{\circ} 64' : 3^{\circ} 64' :: 207^{\circ} 26' : 162^{\circ} 40'$.

XIV.

To find the *Latitude* of any Place, over or near which the Center of any Shadow passes, to any known *Longitude* or *Time* given. And, *vice versa*, To find the *Longitude* or *Time* of the nearest Approach to any such Place to any known *Latitude*. This is no more than proceeding in the Calculations as hitherto ; by taking any known Meridian or Time ; or else any known Latitude for our Examples.

I shall therefore give three several Examples in both Cases ; because of the great Dignity and Usefulness of the Problem : viz. For *Greenwich* the Meridian of the Tables ; for *Dublin* more *Westward* ; and for *Paris* more *Eastward*.

Now I here suppose, from the Calculation and Construction of Eclipses, that the Middle of this general Eclipse will happen May 11. 1724. 17' past 5 a-Clock in the Afternoon ; and that its Center will cross the Meridian of *Greenwich* $41'$ past 6. Upon which Hypothesis I thus compute :

From

From the Angle $100^{\circ} 15' = 6^h 41'$
 Deduct the Primary Angle $40^{\circ} 44'$
 There remains the Angle of the Pole $\angle PC = 59^{\circ} 31'$

Then proceed thus :

R. 10.

CS. $\angle PC$. $59^{\circ} 31'$ 9. 705254

T. P.V. $27^{\circ} 00'$ 9. 707166

T. P.R. = $9. 412420 = 14. 29$

Say then,

CS. P.V. $27^{\circ} 00'$ 9. 949880

CS. P.R. $14. 29$ 9. 985974

CS. V.C. $58. 12$ 9. 719730

19. 705704

CS. R.C. = $9. 755824 = 58. 15$

Deduct R.P. = $14. 29$

Remains P.C. = $40. 46$

Ergo, Lat. $49. 14$

N. B. If we take Dr. Halley's Time $6^h 36^m$
 and subtract $40^{\circ} 44'$ out of 99° there remains
 $58^{\circ} 16'$, and the Calculation will stand thus :

R. 10.

CS. $\angle PC$. $58. 16$ 9. 720958

T. P.V. $27. 0$ 9. 707166

T. P.R. = $9. 428124 = 15. 0$

Then

Then say,

$$\text{CS. PV. 27. } 0. \quad 9. \quad 949880$$

$$\text{CS. P.R. 15. } 0. \quad 9. \quad 984944$$

$$\text{CS. VC. 58. 22. } 9. \quad 719730$$

$$19. \quad 704674$$

$$\text{CS. RC.} = 9. \quad 754794 = 55 \quad 21$$

$$\text{Deduct R.P. 15. } 0$$

$$\text{Rem. P.C. 40 } 21$$

$$\text{Ergo, Lat.} = 49 \quad 39$$

$$\text{Diff. } 00 \quad 25$$

N. B. My Calculation differs from Dr. Halley's Scheme no less than a full Degree of a great Circle, in the Meridian ; if our Difference of Time, which is about 5', be allowed. And though we take the Doctor's own Time, yet do we differ in Latitude 15 Minutes or Miles ; by which Quantity the Doctor's Scheme brings the Center of the Eclipse nearer to London and Greenwich than this Calculation. The reason of which Difference I by no means understand. Time will discover which Determination is most accurate.

Dublin is about 6° 22' Westward in Longitude from Greenwich. Let us find the Latitude of the Center of the Shadow, when it crosses the Meridian of Dublin. We must proceed thus:

As 3.64 to 4.64 so is 6'. 22" to 8'. 7"
Deduct then from the Angle at the Pole used for Greenwich this Difference of these Angles 59° 31' — 8°. 7' = 51° 24'. which is our Angle at the Pole for Dublin. So that if we use the former Figure with that Angle, we compute as before ;

R.

10.

$$\text{CS. 2 P.C. 51. 24. } 9. \quad 795101$$

$$\text{T. PV. 27. } 0. \quad 9. \quad 707166$$

$$\text{T. R.P. } 19. \quad 502267 = 17. \quad 38$$

The

Then say,

CS. PV.	27.	o.	9.	949880
CS. RP.	17.	38.	9.	979099
CS. VC.	58.	22.	9.	719730
			19.	698829
CS. RC.	=	9.	748949	= 55 53
			Deduct RP.	17 38
			Remn ^t P.C.	38 15
			Ergo, Lat.	51 45

Paris is $2^{\circ} 19'$ more *Easterly* than *Greenwich*.

Say therefore $3_{L}64 : 4_{L}64 :: 2^{\circ} 19' : 2^{\circ} 57'$

Now $59^{\circ} 31' + 2^{\circ} 57' = 62^{\circ} 28'$

Then, R. 10.

CS. 2 P.C.	62.	28.	9.	664891
T. P.V.	27.	o.	9.	707166
T. R.P.	=	9.	372057	= 13 15

Say then,

CS. PV	27.	o.	9.	949880
CS. RP	13.	15.	9.	988282
CS. VC.	58.	22.	9.	719730
			19.	708012
CS. RC.			9.	758132 = 55. 03
			Deduct RP	= 13. 15
			Rem ^t .	= P.C. 41. 48
			Ergo, Lat.	= 48. 12

N. B. By such Calculations we may determine the Latitude of the Center of the Shadow's Way, from its entry upon, till its exit out of the Disk of the Earth, to every known Meridian. A Specimen of which I intend to give presently for the several *East* and *West Longitudes* from

from *London* in the Eclipse before us : And another Specimen in the Eclipse, *Sept. 4. 1727.*

If the *Latitudes* be given, as for the Meridian of *Greenwich* $49^{\circ}. 14'$; For *Dublin* $51^{\circ}. 45'$; For *Paris* $48^{\circ}. 12'$: The Case will be that of a Spherical Triangle, when all the Sides are given; and the *Longitude* or *Time* is an Angle sought. Thus in the foregoing Figure for *Greenwich*, $VC = 58^{\circ}. 22'$ is the Side against the Angle sought. $VP = 27^{\circ}. 0'$, and $PC = 40^{\circ}. 46'$. From which data we thus discover the Angle $\angle VCP$.

$$VC = 58^{\circ}. 22' \quad R. 10.$$

$$VP = 27 \quad 00 \quad S. \quad 9. 657047$$

$$PC = 40 \quad 46 \quad S. \quad 9. 814900$$

$$\text{Sum of } 3 = 126 \quad 08 \quad \text{Sum } 9. 471947$$

$$\frac{1}{2} \text{ Sum } 63 \quad 4 \quad S. \quad 9. 950138$$

$$\text{Diff. of } VC \quad 4 \quad 42 \quad S. \quad 8. 913488$$

$$\text{Double Radius} \quad 20.$$

$$\text{The Sum} \quad 38. 863626$$

$$\text{The Remainder} \quad 19. 391679$$

$$\frac{1}{2} \text{ Remainder} \quad 9. 695839 = CS. 60 \quad 14$$

double $120. 28$

from $180. 00$

as before remains $59. 32$

Add the Primary Angle $40. 44$

Sum $100. 16$

Equal in Time to $6^h. 41'$

The Time past Noon of the Center's Transit over the Meridian of *Greenwich*. For

For *Dublin* thus :

$$VC = 58^{\circ} 22' R. 10.$$

$$\begin{array}{rcl} VP & 27^{\circ} 00' & S. \quad 9.657047 \\ PC & 38^{\circ} 15' & S. \quad 9.791756 \end{array}$$

$$\text{Sum of } 3.12337 \text{ Sum } 19.448803$$

$$\begin{array}{rcl} \text{Half Sum} & 61^{\circ} 48' \frac{1}{2} & 9.945159 \\ \text{Diff. of } VC & 3^{\circ} 26' \frac{1}{2} & 8.778383 \end{array}$$

$$\text{Double Radius} \quad 20.$$

$$\text{The Sum} \quad 38.723542$$

$$\text{The Remainder} \quad 19.274739$$

$$\text{Half the Remainder} \quad 9.637369 \quad CS. = 64^{\circ} 18'$$

$$\begin{array}{r} \text{double } 128.36 \\ \text{from } 180.00 \\ \text{remains as before } 51.24 \\ \text{To } 51.24 \\ \text{Add } 40.44 \\ \text{Sum } 92.08 = 6^h 8^{\frac{1}{2}} \end{array}$$

Which deducted from $6^h. 41'$, leaves $32' \frac{1}{2}$ for the Difference of the Angle at the Pole in Time. Say then $46^{\circ} 4' : 36^{\circ} 4' :: 32' 5 : 25' 4$ which is the Difference in time of the Meridians of *Greenwich* and *Dublin*.

[58]

For *Paris* thus:

$$VC = 58 \ 22 \ R. \ 10.$$

$$VP = 27 \ 00 \ S. \ 9.657047$$

$$PC = 41 \ 48 \ S. \ 9.823821$$

$$\text{Sum of } 3 \ 127 \ 10 \quad 19.480868$$

$$\text{Half Sum } 63 \ 35 \quad 9.952105$$

$$\text{Diff. of } VC \ 5 \ 13 \quad 8.958670$$

$$\text{Double Radius} \quad 20.$$

$$\text{Sum} \quad 38.910775$$

$$\text{Remainder} \quad 19.429907$$

$$\text{Half Remainder} \quad 9.714953 = C.S. \ 58.45$$

double 117 30

As before, Remainder 62 30

Add, Primary Angle 40 44

Sum 103 11

In Time 6^h 52¹/₂

From which deduct 6°. 41'. the Remainder 11' ¹/₂ is the Difference of the Angle at the Pole in Time. Say then, 4.64 : 3.64 :: 11 ¹/₂ : 9'. which is the Difference in Time, of the Meridians of *Greenwich* and *Paris*.

Corol. (1.) The latter Branch of the Problem determines the Hour and Minute when the Centre of the Eclipse crosses the Meridian at any assign'd *Latitude*; and by a very small Allowance when the very middle of the Eclipse, or of Total Darknes happens in any Place very near the same.

Corol.

Corollary (2.) The same Branch determines the true *Difference of Meridians* in all Places over or near which the Center of the Eclipse passes, and to the Distance *East* and *West* from any known Meridian whatsoever. Thus because the Meridians of *Paris*, *Greenwich*, and *Dublin* do hence appear to be different as to the Angles of the Pole from the Primary Meridian, in the foregoing Angles, $62^{\circ}. 28'$, $59^{\circ}. 31'$, and $51^{\circ}. 24'$ respectively; It is plain, that the respective *Longitude* of *Paris* and *Greenwich*, when reduc'd from Angles of the Pole to the Difference of Meridians, is $2^{\circ}. 19'$ and that of *Greenwich* and *Dublin* $8^{\circ}. 22'$, and by Consequence of *Paris* and *Dublin* $8^{\circ}. 41'$. Which is no other than the Foundation of my grand Problem of Discovering the *Geographical Longitude of Places by Solar Eclipses*, from the *Latitude given*: Of which more hereafter.

XV.

To find the Duration of a Solar Eclipse, along or near the Path of the Moon's Center, in any Place whatsoever.

From the Motion of the Moon from the Sun gain the Duration of the entire Eclipse; or the Time of that Center's Passage over the Diameter of the Penumbra, if there were no diurnal Motion during that Time, thus:

As the horary Motion of the Moon from the Sun, which is in angular Measure $35^{\circ}. 18'$, and is given in the Calculation; to an Hour or $60'$ in Time :: So let the Diameter of the Penumbra there given, also in angular Measure = $65^{\circ}. 10'$, be to a fourth Number: which will be the Number of Minutes requir'd. In Decimals thus:

$$35^{\circ}. 13' : 60' :: 65^{\circ}. 17' : 110^{\circ}. 8' = 1^{\text{h}}. 50^{\text{m}}.$$

I 2

But

But then, because the diurnal Motion of the Earth must be compounded with this rectilinear Motion of the Moon; and that as we have already stated it, this Eclipse will cross the Meridian of *Greenwich* 41' past 6 a-Clock in the Evening; the one Part of the Duration of the Eclipse being before, and the other Part after that Time, both Intervals must be unequally affected by the diurnal Motion, and we must then take the former half-Duration 55'. 24'' distinctly. And since the diurnal Motion of *Greenwich* is in a larger Parallel than that of the Center of the general Eclipse; while its Obliquity to the Path of the Shadow's Center increases; their respective Motions will nearly keep the same Proportion all along; and so we may safely omit the Consideration of them both. We have also already discovered, that the Velocity of this Center's Motion is here, to that of the Velocity of the diurnal Motion, As 829.6 to 178.4. or as 4.64 to 1. And because the Eclipse begins about 14' before 6, 14' after 6 balances the same; and these 28 Minutes are almost all one, as if there were no diurnal Motion at all. So that we have only 27' 24'' capable of Retardation in the first half: The middle Time of which will be about 30' after 6. We must therefore look into the Table, p. 22, — 26 for the Arc 97°, or its Supplement 82°, corresponding to $89\frac{1}{4}$ of the Sines. Where the Difference of half a Degree is instead of 19.1, as about Noon, no less than 160, which multiplied by 4.64 comes to 742.4. So that the Motion is here retarded a 39th Part. Say then, As 39 to 38, so is 27.4 to 26.7 which 26.7 or 26' 42'' is to be added to the 28 before excepted, for the Duration of the former Part of the Eclipse = 54' 42''. The Middle of the second Part will be about 73 = $180\frac{1}{4}$ backward

ward beyond 90° , that is, $71^\circ \frac{1}{2}$ of the Arc, which corresponds to 85° of the Sines, where the Difference of half a Degree is $61''$; This multiplied by 4.64 gives 285. So that the Retardation is as 19.1 to 285, or 1 to 14.9. Say then, As 14.9 to 13.9; so is 55.4 to 51.7 = $51' 42''$, which is the Duration of the latter Part of the Eclipse, and added to the former Part of the Duration, gives us the whole Duration = $106' 24''$ = $1^h 46' 24''$, without the Consideration of the Elevation of the Luminaries above the Horizon, which a small Matter enlarges that Duration: Of which hereafter.

N. B. If we would be still more precisely nice, we may distinctly allow for the Difference of the Parallels, and the different Obliquity of the Direction; of which p 26, 27. before: Which yet are here omitted, as very inconsiderable.

XVI.

To find the Duration of the Total Darkness along the Path of the Moon's Center; if the Luminaries were in the Horizon.

Say, As the Moon's Motion from the Sun in an Hour, or $60'$; in the Calculation = $35' 18''$ is to those $60'$: So is the Difference of the Diameters of the Sun and Moon in the same Calculation, = $1' 38''$ to a fourth Number: Which will be the entire Duration of the Total Darkness if the Luminaries were not at all elevated above the Horizon. In Decimals thus:

$$35' 13 : 60' :: 1' 62 : 2' 73 = 2' 46'' \frac{1}{2}$$

XVII.

To find the Altitude of the Sun above the Horizon, when the Center of the Shadow crosses the Meridian at *Greenwich*. It

In the Triangle (*Fig. 10.*) ZPS , ZP is = Distance between the Zenith and Pole of the Earth: $38^{\circ} 30'$. PS is = Distance between the Pole of the Earth and Center of the Sun: = Complement of the Sun's Declination, or to $69^{\circ} 28'$. And the Angle ZPS = interval of Time, from the Meridian = $6^h 41' = 100^{\circ} 15'$, whose Supplement is $79^{\circ} 45'$.

Say then,

R.

10.

CS. ZPS $79^{\circ} 45'$ 9. 250280

T. ZP $38^{\circ} 30'$ 9. 900605

T. $PR = 19. 150885 = 8^{\circ} 3' = PR$
 $+ 69^{\circ} 28' = PS$
 $= 77^{\circ} 31' = RS$

And,

CS. PR $8^{\circ} 3'$ 9. 995699

CS. ZP $38^{\circ} 30'$ 9. 893544

CS RS $77^{\circ} 31'$ 9. 334766

19. 228310

CS. $ZS = 9. 232611 = 80^{\circ} 10' = ZS$

Ergo, Altitude = $9^{\circ} 50'$

XVIII.

To find the Azimuth of the Sun at the same Time and Place.

In the former Triangle the Complement of the Altitude being now found = $ZS = 80^{\circ} 10'$. Proceed thus:

S. PS $80^{\circ} 10'$ 9. 993572

S. ZPS $79^{\circ} 45'$ 9. 993013

S. PS $69^{\circ} 28'$ 9. 971493

19. 964506

S. PZS = Suppl. = $9. 970934 = 69^{\circ} 16' = PZ$

From the West $20^{\circ} 44'$

XIX.

XIX.

To find the Number of Minutes that any Solar Eclipse extends to, in a plain Perpendicular to the Axis of the Penumbra.

When the Parallax of the Moon is $57' 17''$, which is near its mean Quantity; the Moon's Distance is 60 Semidiameters of the Earth: And one Second on the Disk of the Earth as viewed at the Distance of the Moon; or on the Disk of the Moon viewed at the Distance of the Earth, is exactly one geographical Mile. And $60''$ or one Minute, is one Degree upon the Disk of the Earth or Moon. But when the Moon's Parallax, as in the Calculation of this Eclipse, is $61' 48''$ whose Sine is $\frac{7926}{100000}$ or $\frac{3515}{100000}$ of the Radius nearly: One Second is less than a Mile; and that in the Proportion of 55.5 to 60. So that about 65 geographical Miles correspond to $60''$, or one Minute: And accordingly, 1781 such Miles will correspond to $32' 35''$, which in that Calculation, is the Semidiameter of the Penumbra, or the utmost perpendicular Extent of the Eclipse.

N. B. We may always make use of the foregoing Tables, pag. 29, — 34 for the perpendicular Distances from the Path of the Moon's Centre, at all Durations of the Eclipses, and of Total Darkness, by still calling the Radius of every *Penumbra* 2000 equal Parts or Miles, and the Radius of every *Umbra* or Total Shadow, 50 such Parts or Miles; and using them accordingly. And this without any other Inconvenience than the Supposition of other than geographical Miles: Which Inconvenience, by the Reduction of them to geographical Miles afterwards, will always, as here, come to nothing.

XX.

XX.

To find the Alteration there is in the Extent and Duration of Eclipses, and of total Darknefs, on Account of the Elevation of the Luminaries at that Time above the Horizon.

This is ever, as the Radius: to the Sine of the Sun's Altitude: As compared with the particular Distance of the Moon at that Time. Thus at the Middle of this Eclipse at *Greenwich*, the Altitude of the Sun has been found to be $9^{\circ}.50'$, whose Sine is $\frac{1728}{5515}$, which divided by 5515 is equal to $\frac{1}{3}$ of the whole: And in the Semidiameter of the *Penumbra*, as well as *Umbra*, amounts to about 6 Miles every Way. Thus also at the Middle of the Eclipse in *North-America*, where 'tis Central at Noon, the Sun will be about 52° above the Horizon: whose Sine is $\frac{788}{5515}$, which divided by 5515 is $\frac{1}{7}$ nearly; which in the same Semidiameters, amounts to about 29 Miles every way. And for the Duration at *Greenwich* add to the common Duration already found, or to $1^h 46' 24''$ the $\frac{1}{17}$ Part of the same, or about $\frac{1}{3}$ of a Minute both ways: which is about $\frac{2}{3}$ in the whole; which will bring the entire Duration from $1^h 46' 24''$ to full $1^h 47'$; and for Total Darknefs will add about $\frac{1}{6}$ of the whole total Darknefs $= 20''$, and increase the Duration from $2' 46\frac{2}{3}$ to $3' 6''\frac{2}{3}$. But for the Alteration in *North America*, which is about $\frac{1}{7}$ both ways, or $\frac{1}{17}$ in the whole; this will there increase the entire Duration $4' 12''$, and from $1^h 46' 24''$ bring it to $1^h 50' 36''$, and for the total Darknefs will add about $\frac{2}{3}$ of the whole, or bring the $2' 46''\frac{2}{3}$ to $4' 47''$.

XXI.

To find the Species and Dimensions of the Ellipsis made by the Total Shadow at the same Places and Altitudes.

The shorter Axis of the Ellipsis, (which is the same with the Diameter of the Conick Shadow it self, in a Plane Perpendicular to the Axis;) is to the Longer, as the Sines before-mentioned. Or at *Greenwich*, as 1708 to 10000; and at *North America*, as 788 to 1000: And since that shorter Axis is in the Calculation 98 Minutes or Miles, of 65 to a Degree; they must be equal to 90 Geographical Miles; which by the last Problem must be increas'd to 96 and to 119 respectively. The golden Rule will therefore soon shew the Length of the longer Axis of the Shadow in both Places.

	sine	rad.	miles	miles
For <i>Greenwich</i>	1708	:	10000	:: 96 : 562.
For <i>North-America</i>	788	:	1000	:: 119 : 151.

XXII.

To determine the central Point in the Elliptick Shadow for *Greenwich*, at the Middle of the same Eclipse.

By *Lemma XIX.* and in its Figure; As VB : to VA :: So is the Sine of $BAV = 9^\circ 34' = 166$: to the Sine of $ABV = 10^\circ 6' = 175$, and so is BC : to AC . And *componendo*, As $166 + 175 = 341$ to 175 :: So is $BC + CA = 562$ to CA , which therefore is equal to 289. Also As $166 + 175 = 341$ to 166 :: So is $BC + CA = 562$ to BC ; which therefore is equal to 274.

For $341 : 175 :: 562 : 289$.

And $341 : 166 :: 562 : 274$.

K

Whence

Whence the Parts of the longer Axis being found, the larger $CA = 289$ Miles: the lesser $BC = 274$ Miles. The half of which, DC is the Distance between the central Point of the Shadow D , and the Center of the Ellipsis C , $= 7\frac{1}{2}$ Miles.

XXIII.

To determine the Angle of Direction of the Total Shadow over the Meridian of *Greenwich*.

This to be done either by Construction, upon drawing the Curve Line of this Motion, through the several Points where, by Calculation, the Center of the Shadow crosses the Meridians of *Paris*, *Greenwich*, and *Dublin*; and measuring the Angle it makes with the several Meridians; or more exactly for *Greenwich* by solving the Triangle (Fig. II.) PVO , where the Arc VO is equally distant *West*, as *Paris* is *East* from *Greenwich*, is to be found by the Method already made use of; thus:

Paris Angle at the Pole $= 62^\circ 28'$. Angle for OV at the Pole is less by twice the Difference of the Angles at the Pole for *Paris* and *Greenwich* $= 5^\circ 54'$, And $62^\circ 28' - 5^\circ 54' = 56^\circ 34'$. First then find the Arc $VO =$ Compl. Latitude for O , as in *Probl. XIV.* by this Analogy.

R.

10.

CS. 2 P C. 56. 34.

9. 741125

T. P V. 27. 00.

9. 707166

T. P R. 19. 448291 = 15 41

Then say

CS. P V.

[67]

CS. PV. 27. 00. 9. 949880

CS. PR. 15. 41. 9. 983523

CS. VG. 58. 22. 9. 719730

19. 703253

CS. PC 9. 753373 = 45. 29.

Deduct 15. 14.

Remt. VO. 39. 48.

VG. 40. 46.

VP. 41. 48.

Then in the Triangle VOP whose included Angle is $4^\circ 38'$; and whose two Sides are $39^\circ 48'$ and $41^\circ 48'$. Find the Arc PR as usual thus :

R.

10.

CS. OVR. 04. 38. 9. 998577.

T. VO. 39. 48. 9. 920733

T. VR. 19. 919310 = 39. 48

from 41. 42

Remnt $2^\circ 6' = PR$.

Then for the Angle VPO , say :

S. PR. 02. 06. 8. 563999

S. VR. 39. 42. 9. 805343.

T. OVR. 04. 38. 8. 908719

18. 714052.

T. VPO. = 10. 150063 = 54. 42.

K 2

And

And for VOR ;

$$\begin{array}{rcl}
 S. VO. & 39. 42. & 9. 805343 \\
 S. VPO. & 54. 42. & 9. 911763 \\
 S. VP. & 41. 48. & 9. 823821 \\
 & & 19. 735584 \\
 S. VOP = & 9. 936241 & = 58. 23 \\
 & & + 113 05 \\
 & & \text{Half } 56 32\frac{1}{2} \\
 & & \text{Compl. } 33 27\frac{1}{2}
 \end{array}$$

So that the Acute Angle VGO , which the Course of the Center of the Shadow makes with the Meridian of *Greenwich*, is $56^{\circ} 32' \frac{1}{2}$; And by consequence the Angle it makes with the Parallel of Latitude there is $33^{\circ} 27' \frac{1}{2}$. 5 or 6 Degrees more than in *Dr. Halley's Scheme*.

Corollary. The longer Axis of the total Shadow at the Meridian of *Greenwich*, makes an Angle of $12^{\circ} 43' \frac{1}{2}$ with the Direction of the Shadow.

For if from the Angle that the Direction of the Center of the Shadow makes with the Parallel of *Greenwich*,

We deduct the Angle that the longer Axis makes with the same Parallel, which is the Azimuth formerly discovered,

There remains this Angle $12. 43\frac{1}{2}$

XXIV.

To Determine the perpendicular Breadth of the Total Shadow, when that Shadow is nearest to *London*; or when the central Point is about the Meridian of *Padstow* in *Cornwall*.

This

This *Problem* is, In an Ellipsis of 96 Miles broad, and 562 Miles long; whose longer Axis makes an Angle with the Direction of the Shadow of $12^{\circ} 43\frac{1}{2}$; To determine the Length of the greatest Perpendicular that falls on that Diameter, along which the Direction of the Motion is. For twice that Perpendicular is the Breadth required. Now I have actually drawn this Ellipsis, or total Shadow, for my Map of this Eclipse. And I find by that Construction, that the Breadth of the total Shadow over *England* is about 155 Miles; or 30 Miles broader than in *Dr. Halley's Scheme*. Time will probably discover which is nearest the Truth.

N. B. This total Shadow increases so greatly in Length as it goes *Eastward*, that it will reach from *Paris* to the utmost Boundary of the Sun-setting at once. And though the Center of the Shadow, by my Calculation, will end $8^{\circ} \frac{1}{2}$ East of the Meridian of *London*, in the Latitude of $45^{\circ} 9'$, or in the *Alps*, not far from *Milan* and *Turin*; Yet by reason of the Extension and Breadth of the total Shadow, and the Refraction of the Rays of Light near the Horizon, it seems probable to me, that all *Switzerland* and *Lombardy*, as far as *Trent*, *Mantua*, *Cremona*, and *Parma*; nay, perhaps, as far as *Venice*, *Padua*, *Bononia*, *Ferrara*, *Ravenna* and *Florence*; may at the same Time be envelop'd in the Total Shadow; and that the Sun may set eclipsed at all or most of those Places at once.

XXV.

To determine the Digits eclipsed, and the Duration of the Eclipse, or of total Darkness, at any

any given Distance from the Path of the Moon's Center; and *vice versa*. These Digits and Durations; if we do not consider the diurnal Motion, are immediately found to any given Distance in the Tables, P. 29, — 34. Thus, if the Digits be 4, or Duration 80' 24", the first Table gives us the Distance of 1350 Miles. And, *vice versa* if the Distance be known to be 1350 Miles, the same Table gives us 4 Digits and 80' 24" Duration. And the like is true of the total Shadow, and the Distance or Duration in the second Table. Thus also we learn from the Map of this Eclipse, that the nearest Approach of the total Shadow to *London* will be about 40 geographical Miles: Which 40 Miles, in this Obliquity of the Motion, is about 31 perpendicular Miles. So that $\frac{31}{114} = \frac{1}{4}$ of a Digit, is the Quantity of the Rim of Light that will be seen at *London*, when the Eclipse is greatest: And by consequence the Digits there eclipsed will be $11\frac{3}{4}$. Nor does it appear to me, that the total Darkness will come nearer to *London*, than 40 Miles; although Dr. *Halley's* Scheme brings it within 7 or 8 Miles. As for the Duration of the entire Eclipse here, it will be much the same as if it were Central: And has been already determined to $1^h 47'$. And for the total Darkness along the Path of the Center between *Exeter* and *Plimouth*, its Duration will be, as before stated, $3' 6'' \frac{1}{2}$.

N. B. As to the Alterations which will arise in more considerable Distances from the Path of the Moon's Center, proceed thus: First, find the Minutes of Duration corresponding to any given Distance in Miles, *e.g.* 1350, thus; $2000:1350::32' 63:22' 02$. Or, As 2000 Miles, the Radius: to 1350 the Sine :: So are the Number of Miles

Minutes for the Semidiameter of the Penumbra :
to the Number of Minutes for that Sine. Now,
32' 19" the present Length of the greatest Di-
stance — 22' 01" = 10' 18". Then compute
every Thing as if the Center of the Shadow were
10' 18", Latitude or nearest Distance ; and
as if the Chord of that Arc over the Earth's Disk
were the Path of the Moon's Center : and the
Chord of the Distance from the Center of the
Penumbra were the Diameter of the Penumbra :
And all will be discovered by the Rules before go-
ing ; without any new Directions whatsoever.
Only if this Distance be the contrary way to the
present Example, we must here make use of Ad-
dition instead of Subtraction.

N. B. Upon consulting the *French Ephemerides*
of *Des Places*, recommended, in some Degree,
by *Cassini* himself, I find that he determines the
Eclipse at *Paris* thus.

Beginning, May 22. (N. S.)	06 03 00
	Middle 06 58 38
	End 07 54 00

Whence the Duration there is 01 51 00
Digits eclips'd 10 $\frac{1}{2}$

Latitude of the Moon at the Middle 39' $\frac{1}{2}$

Difference from me in Digits 01 $\frac{1}{2}$

In Latitude 07 10

If these Digits and this Latitude be right, Dr.
Halley, and I, and those *English Astronomers* that
have computed and constructed this Eclipse, who
all, I think, do agree, that it will be total at *Pa-
ris*, and that for two or three Minutes also, are
prodigiously mistaken. For if *Des Places* be in
the right, *Copenhagen* and *Stockholm* will stand
much fairer for the Pretence to a total Eclipse,
than

than either *Dublin*, *London*, or *Paris*. Time will certainly determine who are in the right.

N. B. If we add the Duration of the Eclipse, consider'd without Regard to the diurnal Motion, to the Time of the Center of the Shadows passing over the entire Disk, we gain the entire Duration of the Eclipse in general, thus :

	h	l	''
To	1	50	48
Add	2	58	24
Sum	4	49	12

Therefore the general Eclipse by the Meridian of *London*,

	h	l	''
Begins II.	52	24	
Middle V.	17	00	
End VII.	41	36	
Duration 4 ^h	49	12	

Eclipse at *London*,

Begins V.	45	00
Middle VI.	41	00
End VII.	32½	00
Duration 1 ^h	47½	00
Digits eclipsed II	4½	

Duration of Total Darkness between }
Exeter and *Plimouth*, } 03 6½

N. B. Dr. *Halley's* Times are still about 5' sooner than mine, and his general Duration about 7', and his Duration at *London* about 1' longer.

N. B.

N. B. I cannot tell the Reason why my Original Calculation of this general Eclipse, which has been carefully made according to Sir *Isaac Newton's* famous *Theory of the Moon*, does here so much differ from Dr. *Halley's* Determination, as 5' in Time; especially since both those Methods did very well agree, in the last celebrated total Eclipse, *Apr. 22. 1715.* 'Tis Time alone that can determine between these two Methods of Calculation.

N. B. I have lately been shewed an exact Scheme of this next Eclipse, according to Mr. *Flamsteed's* own Tables and Determination, and made in his Life-time; wherein the Digits eclipsed are $11\frac{8}{11}$ exactly according to my Determination in this Paper.

L

A P R O-

A P R O P O S A L

For the Discovery of the L O N G I -
T U D E of the several Places of
the Earth , by Total Eclipses of
the Sun.

IT is humbly proposed, That Observations be made in all Places where Solar Eclipses are seen, of the exact *Duration* of the same ; by either viewing the Beginning and Ending thereof through a Telescope, with a Glass smoaked in the Flame of a Candle, for saving the Eye of the Observer ; or else by casting the Sun's Image through such a Telescope upon white Paper, and viewing the first and last Impression of the Moon's Shadow upon it. And that the Hour, Minute, and Second of some Pendulum Clock be carefully noted at the same Time : And that when the same Observations are transmitted for the Uses of Geography, the Latitudes of the Places be also set down and transmitted at the same Time : That the like Observations be also made in all Places where such Eclipses are total and visible, of the exact Beginning and Ending, with the Duration of Total Darknes ; by the like Comparison of a Pendulum Clock , or other pendulous Body that vibrates Seconds, or half Seconds; and, with the Latitude be transmitted in the same Manner, and for the same Uses. How by the Help of these
Ob-

Observations the *Longitude* may be discovered, I have already shew'd under *Problems* XIV, XV, and XXV. foregoing.

N. B. If the *Latitude* be exact, and the entire Duration of an Eclipse of the Sun can be determined to one Quarter of a Minute in Time ; and the Times of Beginning and Ending be not too remote from Noon ; the *Longitude* of that Place may usually be thence determined much nearer than to one Degree. And since 'tis certainly very practicable to observe the Duration of Total Darkness to three or four Seconds every where ; if the *Latitudes* be exact, the *Longitude* of those Places may commonly be determined much nearer than a Quarter of a Degree. Nor will the grosser Way by the Duration of the entire Eclipse, unless the Beginning or Ending be near the Horizon, be much inferior, as to Accuracy, to any Method now used by Astronomers for that Purpose ; while it is withal much more easy and practicable, even by those that understand little or no Astronomy, or that have no curious Astronomical Instruments, than any of them. And as for all such Places as lie in the Trace of the Total Shadow, 'tis evident, that no other Way now used is in the least comparable to this before us ; either in Ease of Observation, or Accuracy of Determination therefrom.

N. B. Since this next total Eclipse is so near the Evening, and the Motion of no small Part of its Shadow's Passage over *North America*, and the *North-West* Parts of *Europe* nearly parallel to the Equator ; the general Eclipse is much less fit for such Observations, as are here necessary, than most others of the same Sort ; though where a total Shadow passes, no Eclipse can be wholly unfit for

it. However, if this be less favourable to us in general; the other which I have brought to Calculation also, 1727. is exceeding favourable; as not going very far over the *West of Africa*, 'till a good while in the Day; as going very oblique to the Parallels almost all the Way; and as reaching the *East-Indies* long before the Sun comes to the Horizon: On all which Accounts I venture to recommend it for that Purpose, and to say, That in case the Clouds generally favour us, and a sufficient Number of accurate and proper Observations be not wanting, the Geographical Longitude of Places in no small Part of the one half of the known World, may by that single Eclipse be very well determined.

N. B. The Calculations here made before-hand, with regard to both these Eclipses, are not themselves design'd for the actual Discovery of that Geographick Longitude of Places immediately; but only as Specimens how such Calculations may and ought to be more exactly made after the Eclipses are over, and their true State, and Time and Circumstances well known from the best Observations, but not otherwise.

N. B. If any curious Persons have a mind to examine very nicely any of the Eclipses which will be visible at *London* till A. D. 1740. they are these following; according to the Calculation of Mr. *Leadbetter*.

SOLAR ECLIPSES.

(1.) A Total Eclipse, *May 11. 1724. 36' past Six a-Clock at Night, of 11½ Digits at London.*

(2.) An Eclipse, *Sept. 14. 1726. 38' past Five a-Clock in the Evening, of 6 Digits.*

(3.) An

(3.) An Eclipse, *Sept.* 4. 1727. 27' past Seven in the Morning, of $2\frac{1}{2}$ Digits.

(4.) An Eclipse, *July* 4. 1730. 26' past Nine in the Morning, of 6 Digits.

(5.) A great Eclipse, *May* 2. 1733. 36' past Six at Night, of $9\frac{1}{3}$ Digits.

(6.) A great Eclipse, *Feb.* 18. 1737. 25' past Three in the Afternoon, of $9\frac{1}{4}$ Digits.

(7.) An Eclipse, *Aug.* 4. 1738. 58' past Nine in the Morning, of $4\frac{1}{2}$ Digits.

(8.) A considerable Eclipse, *July* 24. 1739. 23' past Four in the Afternoon, of $7\frac{1}{12}$ Digits.

(9.) An Eclipse, *Decemb.* 19. 1739. 49' past Eight in the Morning, of $2\frac{1}{2}$ Digits.

L U N A R E C L I P S E S.

(1.) An Eclipse, *Octob.* 21. 1724. 9' past Four in the Morning, of 7 Digits.

(2.) A great and total Eclipse, *Octob.* 10. 1725. 15' past Seven at Night, of $21\frac{1}{2}$ Digits.

(3.) An Eclipse, *Sept.* 30. 1726. 4' past Five in the Morning, of 5 Digits.

(4.) An Eclipse, *Feb.* 14. 1728. 14' past Seven in the Morning, of $9\frac{1}{2}$ Digits.

(5.) A great and total Eclipse, *Feb.* 2. 1729. 44' past Eight at Night, of $19\frac{1}{2}$ Digits.

(6.) A great and total Eclipse, *July* 29. 1729. 20' past one in the Morning, of $18\frac{1}{2}$ Digits.

(7.) An Eclipse, *Jan.* 23. 1730. 34' past Three in the Morning, of $2\frac{1}{2}$ Digits.

(8.) An Eclipse, *June* 9. 1731. 55' past One in the Morning, of $1\frac{1}{2}$ Digits.

(9.) A great and total Eclipse, *Novem.* 20. 1732. 59' past Nine at Night, of $20\frac{1}{2}$ Digits.

(10.) An Eclipse, *May* 17. 1733. 56' past Six at Night, of $2\frac{1}{12}$ Digits.

(11.) An

- (11.) An Eclipse, *Sept.* 21. 1735. 33' past One in the Morning, of $5\frac{1}{2}$ Digits.
 - (12.) A great and total Eclipse, *March*, 15. 1736. 52' past Eleven at Night, of $21\frac{1}{2}$ Digits.
 - (13.) A great and total Eclipse, *Sept.* 6. 1736. 2' past Three in the Morning, of 20 Digits.
 - (14.) An Eclipse, *Aug.* 29. 1737. 44' past Three in the Morning, of $4\frac{1}{2}$ Digits.
 - (15.) An Eclipse, *Jan.* 13. 1739. 47' past Ten at Night, of $6\frac{1}{2}$ Digits.
 - (16.) A great and total Eclipse, *Jan.* 2. 1740. 45' past Ten at Night, of 20 Digits.
 - (17.) An Eclipse, *Decemb.* 21. 1740. 49' past Ten at Night, of $5\frac{1}{2}$ Digits.
-

Advertisement.

Mr. *Whiston* Proposes to make an *Orery* for the Explication of the Motions of all the Planets and Comets; and a *Copernicus Improv'd*, for the easy Discovery of all Solar Eclipses whatsoever, from the Beginning of Astronomy, till A. D. 2000. and for the natural and universal Exhibition of the same to the Eye in a few Minutes time, and this with a Globe of 30 Inches Diameter. Price of both 20 Guineas.

A Table of the Latitude of the Center of the total Shadow at the Eclipse, May 11. 1724. to every 10 Degrees of the Angle about the Pole, and every 7^o 845. Longitude from London.

Angles at the Pole.		Longitude from London.	Latitude.	
°	'		°	'
70	19	8 ^o 577	45	9
69	23	7 ^o 845	45	34
		West.		
59	23	0000	49	17
49	23	7 ^o 845	52	19
39	23	15 ^o 690	54	42
29	23	23 ^o 535	56	29
19	23	31 ^o 380	57	43
09	23	39 ^o 225	58	13
00	00	46 ^o 600	58	38
00	37	47 ^o 1070	58	38
10	37	54 ^o 915	58	21
20	37	62 ^o 760	57	36
30	37	70 ^o 605	56	18
40	37	78 ^o 450	54	27
50	37	86 ^o 295	51	59
56	22	90 ^o 021	50	16
60	37	94 ^o 140	48	51
70	37	101 ^o 985	45	5
80	37	109 ^o 830	40	40
90	37	117 ^o 675	35	56
100	37	125 ^o 520	30	31
110	37	133 ^o 365	25	15
120	37	141 ^o 210	20	11
130	37	149 ^o 055	15	37
136	59	154 ^o 052	13	3

Mid*

N. B. Here, as well as in the next Table, The Angles at the Pole differ by Ten Degrees; and the Longitude from London is found by the Proportion of 4164 to 3164, to be 7^o 845 of Longitude from

from London for every such Ten Degrees. But if, as is generally most convenient, we would have the Differences of Longitude from London be even 10 Degrees; We must find the correspondent Angles at the Pole before we begin our Calculations by the inverse Proportion of 3, 64; 4, 64 :: 10°: 12½ the perpetual Addition of which Number will give us a Series for such a Calculation.

A Table of the Latitude of the Center of the total Shadow at the Eclipse, Sept. 4. 1717. to every 10 Degrees of the Angle at the Pole; and every 7° 167 Longitude from London.

Angles at the Pole.		Longitude from London.		Latitude.
0 1		West.		North
11 31		12,248		30 18
4 28		7,167		30 20
		East.		
00 00		3,963		30 14
5 32		0,000		29 28
15 34		7,167		27 56
25 32		14,334		25 40
35 32		21,501		22 38
45 34		28,668		18 27
55 34		35,835		14 24
65 32		43,002		9 21
75 32		50,169		5 4
77 58	Midle.	51,903	Midle.	3 57
85 32		57,336		0 16
				South.
95 32		64,503		4 32
105 32		71,670		9 55
115 32		78,837		14 54
125 32		86,004		19 16
135 32		193,171		23 00
145 32		106,338		25 57
155 32		101,505		28 5
165 32		106,672		29 34
168 39		116,909		30 10

3,53 : 2,53 :: 10 : 7,167.

2,53 : 3,53 :: 10 : 13,952.

A Cal-

A Calculation of the Total Eclipse of the Sun, A.D. 1724.

May 11. Post Meridian.

	M.'s mean Mo.	Moti ⁿ of Apogee.	Moti ⁿ of Node r.r.
<i>Annus Dom.</i> 1701	10 15 20 60	11 8 21 00	4 27 34 30
Years 20	4 15 34 05	3 3 50 15	26 50 15
and 3	28 9 10	4 1 59 32	1 27 59 9
Leap-Y. May, Days 11	9 29 17 03	14 48 21	6 59 14
Hours 5	2 44 42	2 24	40
Minutes 15	8 14	4	2
(Mean Time.)			3 1 29 30
Moon's mean Motion	1 29 13 14	6 28 53 36	1 25 34 50
Sun's mean Anomal.	10 22 19 57	An. Eq. add 12 2	An. Eq. subtr. 5 43
Physic. Parts subtr.	7 7	6 29 5 38	1 25 29 7
1d Equat. subtr.	2 31 5	Mean Pla. correct	Mean Pla. correct
3d Equat. add	10 1	2 1 38 45	2 1 38 45
4th Equat. add	2 6	2 16	Sun's true Place
Diff. subtr.	7 22	7 2 33, 7	6 9 38
Moon's mean Pl correct	1 29 5 52	Annual Arg. ment	S's diff. from Node
True Pl of Apog. subtr.	7 9 14 58	10 9 0	Equat. add 18 56
Moon's mean Anomal.	6 19 51 24	Equat. add	1 25 48 3
Equat. add	2 31 7	7 9 14 38	True Pl. of Node
Moon's true Pl. in Orbis	2 1 36 59	True Pl. of Apogee	5 17 9
Sun's true Pl. subtr.	2 1 38 45	Eccentricity 6000	Inclinat. Limit.
Moon's diff. from Sun	11 29 58 14		
Nodes true Pl. subtr.	1 25 48 3	M's h. mot. 37 42	S. mid. Sun 15 53
Argument of Latitude	5 48 76	Sun's 2 24	Sem Moon 16 42
Reduction subtr.	1 30	M's fr. Sun 35 18	Sem Pen ²² 32 35
Moon's tr. Pl. in Eclip.	2 1 35 29	35 18 : 60 :: 1	Semi Diff. 61 38
Moon's true North Lat.	32 14 46	Angle of the M's	Way with the
At the Eclipse	32 19	Eq. time add 3 53	Ecliptick 5 36
		Reduct. subtr. 2 33	Diff. Diam ^r of
		Diff. add 4 22	of the Sun & Moon,
		Erge, Eclipse is	or Br. adth of
		119 5 ^h 19 42"	total Shad. 0 98
		Temp. appar.	
		deduct the Error	
		of the Tables a-	
		bout 2 22"	
		true Tim. 5 ^h 17 0	

A Calculation of the Sun's Place for the same Time.

	Sun's mean Motion.	Motion of Perihelion.
<i>A. D.</i> 1701	9 20 43 40	3 7 44 30
Years 20	9 4	26 0
And 3	11 29 17 0	3 4
Leap-Y. May, Days 11	4 10 6 19	23
Hours 5	12 10	
Minutes 15	37	3 8 9 2
(mean Time)		Place of Perihelion
Sun's mean Motion	2 0 28 59	2 00 28 59
Equation add	1 9 46	Moon's mean Motion
Sun's true Place	2 1 38 45	10 22 19 57
		Sun's mean Anom.

**A Calculation of the Total Eclipse of the Sun, A. D. 1727.
Sept. 4. Mane.**

	M's mean Mot.	Motion Apogee.	Mot ⁿ of N ^o 2 ^d retr.
	1 0 1 1	1 0 1 1	1 0 1 1
<i>Ann. Dem.</i> 1701	10 15 20 0	11 8 20 0	4 27 24 20
<i>Years</i> 20	4 13 34 5	3 3 50 15	26 50 15
<i>And</i> 6	2 9 28 35	8 4 5 44	3 26 1 29
<i>Sept. Days</i> 3	5 23 35	27 24 23	13 1 37
<i>Hours</i> 20	10 58 49	5 34	2 39
<i>Minutes</i> 35	19 13	10	5
(Mean Time.)			5 5 56 5
Moon's mean Motion	5 21 4 37	11 13 46 6	11 21 28 15
Sun's mean Anomaly	2 15 32 16	An. Eq. sub. 18 59	An. Eq. add 9 7
Physical Parts add	11 23	11 13 27 7	11 21 37 22
ad Equat. subtrac ^t	0 38	Mean Pl. Apo. cor.	Mean Pl. of No. 01
6th Equat. add	1 57	5 21 52 53	5 21 52 53
Difference add	12 42	Sun's true Place	Sun's true Place
Moon's mean Pl. correct	5 21 17 19	6 8 25 40	6 0 15 31
True Pl. of Apog. subtr.	11 16 23 44	Annual Argum.	5.2's dist. fro. Nod.
Moon's mean Anomaly	6 4 53 35	Eq. Ad 2 56 27	Eq. Ad. 0 48
Equation add	42 26	11 16 23 44	11 21 38 10
M.'s Correct Pl. in Orb.	5 21 59 45	True Pl. of Apo.	True Pl. of Node
Sun's true Pl. subtrac ^t	5 21 52 53	Eccentr. 66337	5 17 20
Moon's Dist. from Sun	6 53		Inclin. Lim't.
Variation add		9 S's hor. mot. 2 26	
M.'s true Pl. in Orbit	5 21 52 56	Moon's 38 29	Semid. S. 16 1
Nodes tr. Pl. subtrac ^t	11 21 38 10	M. fr. S. 36 2	Semid. M. 16 50
Argument of Latitude	6 0 21 44	36 04 : 60 : 2	Diff. = 00 49
Reduction subtrac ^t		6 19 : 11 : 30	Sum = 33 52
Moon's true Pl. in Eclips.	5 21 51 48	Ergo, Eclipse will	Parallax M. Ho-
Moon's true <i>Sensu</i> Latit.	1 54	be 20 ^h 23 34	rizon 62 7
At the Ecliptic	1 23	Mean Time.	Sun 30
		Eq. tim. ad. 4 38	Semid. D'f 61 57
		Reduct. subtr.	Angle of the M.'s
		Diff. add 4 29	visible Way with
		20 ^h 28 03	the Ecliptick
		Appar. Time.	5 36 20

A Calculation of the Sun's Place for the same Time.

	Sun's mean Motion.	Motion of Perihel.
	1 0 1 1	1 0 1 1
<i>A. D.</i> 1701	9 20 43 40	3 7 44 30
<i>Years</i> 20	9 4	21 0
<i>And</i> 6	11 21 33 8	6 18
<i>Sept. Days</i> 3	8 2 28 9	40
<i>Hours</i> 20	49 17	
<i>Minutes</i> 35	1 26	
(Mean Time.)		3 8 12 28
Sun's mean Motion	5 23 44 44	Place of Perihel.
Equat. subtrac ^t	1 51 51	Sun's mean Motion
Sun's true Place	5 21 52 53	2 15 32 16
		Sun's mean Anomaly

Some Account of Observations lately made with Dipping-Needles, in Order to discover the LONGITUDE and LATITUDE at Sea.

UPON the Receipt of the liberal Assistance of His most Excellent Majesty, King GEORGE; their Royal Highnesses the Prince and Princess of *Wales*, and many other of the Nobility and Gentry, my kind Friends, I sent last Year Four several Dipping-Needles to Sea; with Frames hung near the Center of Motion in Gimbols, to avoid the Shaking of the Ship; and with proper Instructions to the Masters of the Vessels: And this, in order to discover the State of Magnetism in the several Parts of the Globe; and to find whether accurate Observations could be made at Sea, and to determine whether the fundamental Theory I laid down from former Observations would hold or not; viz. "That Magnetick Variation and Dip
" are all deriv'd from one Spherical Magnet in
" the Center of our Earth; with an irregular
" Alteration of the Variation, according to the
" different Degrees of Strength of the several
" Parts of the Loadstone, as compounded with a
" very slow Revolution from *East* to *West*: And
" with a regular Alteration of the Dip, nearly
" according to the Line of Sines, from the Magnetick Pole to the Magnetick Equator; the
" Axis of that Equator being sufficiently Oblique

“ to its Plane : All which is the Case of Spherical Loadstones here.” Now having already received Four Journals from Four, several Masters employ’d, I take this Occasion of returning my Benefactors hearty Thanks for their Assistance, and of giving them and the Publick some Account of the Success of these Observations, and what Consequences are naturally to be drawn from them ; with the Difficulty hitherto met with in the Practice at Sea , and the proper Remedy for the same in future Trials.

Captain *James Jolly* set out in *July*, 1722. for *Archangel*, with one of my Dipping-Needles on Board. He, for some time, met with such Difficulties in the Practice, as confin’d to the Frame I had given him , that he was not at first able to make any good Observations at all. But after some Time, he took the Needle into his own Cabin ; and without any Approach to the Center of Motion, or any Contrivance for avoiding the Shaking of the Ship at all ; having a clear and full Gale all along, but without any stormy Weather , He made me 28 very good Horizontal Observations, from the Latitude of 65 quite to *Archangel* : I say, *Horizontal Observations* only, as I desired him ; the Needle, by an Accident before he went, being rendred incapable of making any other with sufficient Accuracy. In this Space the Needle altered its Velocity very greatly, as I expected it would : And 5 Vibrations which at first were perform’d in about 280'', beyond the *North-Cape* came to 250'', till towards *Archangel* it gradually returned to about 177''.

Captain *Othniel Deal* set out about the same Time for *Boston* in *New-England*, with the same Instru-

Instrument, and made Four Observations of the Dip, both by the Vertical and Horizontal Vibrations, and by the Dip it self; Three upon the open Sea, and One in the Haven of *Boston*: Which in some small Manner differed one from another, but in the main agreed, and kept the due Analogy I expected. He greatly complained of the Shaking of the Ship, till in *Boston* Haven he made a nice Observation both Ways, which did not greatly differ: Tho' the greatest Part of of his Observations by the Dip it self were somewhat more agreeable to Analogy than the other. The Reason was, I take it, that, as he assured me, he always took great Care to avoid the Shaking of my Frame; which Frame tho' it very much avoided the slower and greater Oscillation of the Ship, yet made a quicker but lesser Oscillation it self: Which Fault I was sufficiently sensible of just before the Ships were going away, but was not able then to obviate; as I am prepared to do hereafter. After Captain *Beal* had made and sent me these Observations, he pursued his Voyage to *Barbados*, and thence to *Charles Town* in *South Carolina*; at both which Places he made Observations; but the best at *Barbados*. For before he came to *Carolina*, he observed the Axis of the Needle to shake; which made him take the Dip there otherwise than he ought to have done; which is the natural Occasion that the Dip there did not so well agree to Analogy as the rest. However, upon my Receipt of his first Journal, with the Four first Observations, especially the exact one at *Boston*, I formed a more exact Theory of the Proportion of the Alteration of the Dip in the Spherical Magnet of the Earth; and found it at this Distance of the Earth's Surface, not far from that in my Spherical Loadstone, at the Distance

stance of about $\frac{3}{8}$ of an Inch from its Surface ; viz. Not exactly as the Line of Sines, where at the Middle of the Line the Angles are 60 and 30; but rather as 66 to 24. Which Rule therefore is what I now propose as much nearer than the other. By which Proportion I determined long before-hand the Dip at *Barbados* of 43° or 44° , as many of my Friends can witness: And when Captain *Beal* delivered me the Paper of this Observation at *Barbados*, before I opened it, or in the least knew what Dip it contain'd, I foretold to him from that Theory the very same Dip, which both himself and his Paper immediately assur'd me to be true; and whose Truth, as he inform'd me afterwards, was confirm'd by another Observation, made a little before in the open Sea, of about 45° .

Captain *Tempest* also, about the same Time, set out for *Aniegoa* and *St. Christopher's*, with the same Instrument and Frame. In his Letter, dated last *January*, he greatly complains of the Shaking of my Frame; and proposes an Hint how it might be avoided: Which Method of its Avoidance I had long before thought of, and provided for accordingly; and which has been a full Year ready for Practice. Those Observations of his, that I have yet received; for I have not heard from him since *January*, but hope soon to here farther; were but Three, and all at open Sea; and but one of them made both the Ways that I desired: And, indeed, seem the least agreeable to Analogy of any of the rest. Only since that single Observation, which was also made by the horizontal Vibrations and vertical Oscillations, agrees very well to that Analogy; since they all three are about the same

Quan-

Quantity of 8 or 9 Degrees exceed that Analogy ; and since very near the same Place, where the third Observation was made, I have a double Observation of Captain *Beal's* to correct the same ; I rather conclude, that Captain *Tempest* made a Mistake, and placed the wrong Edge of the Needle upward in all the Three Observations : Which would naturally occasion such a Difference. When I receive the rest of his Observations, or his Needle again, I shall be able to judge better of that Matter. However, even these Observations agree in gross with all the rest, to the gradual Decrease of the Dip as you go nearer to the Equator : Tho' as they stand at present, they do not determine the accurate Proportion of that Alteration so well as the others.

Captain *Michel* also, long after the rest, set out for *Hamburgh* with the same Instrument ; though now without the Frame, which he was not willing to incumber himself with ; and I suspected that in its present Contrivance it did more hurt than help the Nicety of the Experiments. I also by him, sent a Letter to the Reverend Mr. *Eberhard*, who was the Occasion of my studying this Matter, and was then Pastor of *Altena*, close by *Hamburgh* ; desiring that he would there make the Experiment very exactly, and give me a particular Account of it. But I have not yet received his Answer.

Now the Observations here mentioned, as well as those many others I had by me before, do seem to me in general evidently to afford us the following Inferences :

(1.) That there is one Spherical Loadstone, and but one in the Center of our Earth ; and that this Load-

Loadstone, like other Spherical Loadstones, has but one *Northern Pole* : Contrary to Dr. *Halley's* Hypothesis.

(2) That this *Northern Pole* is situated, contrary to the same Hypothesis also, a great Way to the *East* of our Meridian : And indeed, as I before had determin'd, about the Middle of the Distance between the *North Cape* and *Nova Zembla*. Captain *Jolly's* numerous Observations prove this most fully : While in Sailing towards that Point his horizontal Vibrations greatly increas'd in Number : And when he turn'd almost a right Angles, as he went down to *Archangel*, they soon diminish'd ; and yet so little, after some time that it was evident he then sail'd not far from a Parallel to that *Northern Pole* ; and not very many Degrees from it neither ; exactly according to my Expectations.

(3) That the absolute Power of the internal Magnet is considerably different in different Places ; and that without any certain Rule ; as it is upon the Surface of our *Terrella* or Spherical Loadstones here. This the various Number of Seconds to a vertical Oscillations, and all the Accounts in the other Observations fully prove ; and by consequence this must cause different Variations in different Places, as is the Case of our *Terrella*.

(4) That there no where appears in open Seas any such Irregularity in the Dip, as we sometimes meet with near Shores, or at Land ; and by consequence that Dr. *Halley's* grand Objection against the Discovery of the Longitude by the Dipping-Needle, taken from an Observation of his own, concerning such an Irregularity near the Shore at *Cape Verd* ; and from his own Hypothesis of the four Magnetick Poles is utterly groundless.

less. Nor indeed shall I be at Rest, till I have sent a Dipping-Needle to *Hudson's Bay*, on purpose to determine this Dispute about the four Poles; For that Voyage being almost directly towards his second Northern Pole all the way, and about the same Distance all the way from mine; if this Voyage afford much the same Dip, it will demonstrate that there is but One Northern Pole; and that it is nearly where I place it. But if that Dip greatly increase, it will demonstrate a second Pole somewhere in those Parts of *America*, where Dr. *Halley* places it. And to this Decretory Experiment do I appeal for a final Determination of this Question. The Doctor seems to me to draw his Inferences from the Variation, which no Way proves any such double Poles, as being still as sensible on our *Terrelle*, which have no more than single ones; while he avoids all Observations from the Dip, which are still against him; and which are alone capable of discovering the exact Place of such Poles, either upon the Surface of the Earth, or of *Terrelle*. However, when one Set of Experiments with a Dipping-Needle, sent to *Hudson's Bay*, will certainly determine this Matter, 'tis a vain Thing to go on in the Way of Controversy about it.

In short, The Observations hitherto made, shew that the Foundations I go upon in this Discovery of the Longitude and the Latitude at Sea, are true and right: That the Terrestrial Magnetism is very regular and uniform, in the open Seas; that the Latitude in the Northern Parts may even, without any Avoidance of the Shaking of the Ship, in ordinary calm Weather, be in good Degree thereby discovered already; and that if I can sufficiently avoid the Shaking of the Ship, which I am now endeavouring, and have great

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Hopes

Hopes of performing, both *Latitude* and *Longitude* may by this Method be discovered in the greatest Part of the sailing World: I say nothing here of another Method of Trial, which I am also pursuing, and which depends, like this, on the avoiding the main Part of the Ship's Agitation; and if effected will be more easy and universal than this. But as to giving any farther Account of that to the Publick, unless it succeed, I have no Intention at all.

N. B. The original Journals are all in the Hands of my great Friend and Patron *Samuel Molyneux*, Esq; Secretary to his Royal Highness, the Prince of *Wales*, and Fellow of the Royal Society: Which Journals, when I have compleated the rest of the Observations I hope to procure, I intend to publish entire, for the more full Satisfaction of the curious,

A Table of the Angle of Inclination below the Horizon, in Dipping-Needles, to every Part of their respective equal Distances from the Magnetick Poles and Equator.

Dist. from the Pole.	Dip.	Dist. from the Equat	Dip.
0	• •	0	• •
1	89 30	1	08 41
2	89 00	2	12 25
3	88 27	3	15 14
4	87 59	4	17 41
5	87 29	5	19 51
6	86 59	6	21 50
7	86 38	7	23 41
8	85 58	8	25 24
9	85 27	9	27 2
10	84 57	10	28 36
11	84 27	11	30 6
12	83 56	12	31 32
13	83 26	13	32 55

Di-

Dist. from
the Pole.

Dip.

•	•
14	82 55
15	82 24
16	81 54
17	81 23
18	80 52
19	80 21
20	79 49
21	79 18
22	78 47
23	78 16
24	77 44
25	77 12
26	76 41
27	76 8
28	75 36
29	75 4
30	74 32
31	73 59
32	73 26
33	72 54
34	72 20
35	71 47
36	71 14
37	70 39
38	70 5
39	69 31
40	68 57
41	68 22
42	67 47
43	67 12
44	66 36
45	66 00

Dist. from
the Pole.

Dip.

•	•
14	34 15
15	35 33
16	36 50
17	38 4
18	39 16
19	40 28
20	41 37
21	42 45
22	43 54
23	44 58
24	46 2
25	47 6
26	48 9
27	49 10
28	50 12
29	51 12
30	52 12
31	53 11
32	54 9
33	55 6
34	56 1
35	57 0
36	57 56
37	58 52
38	59 47
39	60 41
40	61 35
41	62 49
42	63 22
43	64 15
44	65 8
45	66 0

N 2

N. B.

N. B. I take the Northern Pole of the Terrestrial Magnet to be about the Meridian of *Archangel*, in the Latitude of 75° . Its Equator to be nearly a great Circle, intersecting the Earth's Equator about 2° Degrees Eastward of the Meridian of *London*; and in its opposite Point. And that its utmost Latitude Northward is in the Gulph of *Bengall* about 12° Degrees; and as much South in the opposite Point, in the great South Sea. And that the Southern Pole is nearly circular; its Radius 40 Degrees of a great Circle, and its Center in a Meridian Eastward from *Ceylon* about 4° Degrees, and about 68° Latitude.

N. B. *London* is nearly $\frac{24}{33} = \frac{1}{3}$ distance from the North Pole of the Magnet, whence its Dip will be 74° , which is certainly so in Fact. *Boston* in *New-England* is $\frac{11}{17} = \frac{1}{2}$, whence its Dip will be about $68^{\circ} 22'$, which Captain *Beal* found to be so in Fact. *Barbados* is about $\frac{16}{33} = \frac{2}{3}$ distant from the Equator of the Magnet, whence its Dip ought to be about 44° , as Captain *Beal* also found it to be in Fact. *St. Helena* is about $\frac{14}{29} = \frac{2}{3}$, whence its Dip ought to be about $47^{\circ} 50'$ as Dr. *Halley* found it to be in Fact. And so every where in the main Ocean, at considerable Distances from the Shores.

N. B. If the Dip of any Needles be somewhat different at *London*, add or subtract a proportionable Part of the Dip elsewhere. And you will have nearly the true Dip at any other Place with that Needle. Thus if your Needle differ from the other 2° or $120'$, and shew the Dip at *London* $72^{\circ} 45'$ instead of $74^{\circ} 45'$, which is its proper Dip in this Table; and you require the true Dip by this Needle for

Boston in *New-England*; *Southward*; which in the Table is $68^{\circ} 22'$; proceed thus. Because the equal Distance of *Boston* from the Magnetick Equator is 49 Parts of 60, 4, the like Distance of *London* from that Equator; deduct $120' = 2^{\circ} = 1^{\circ} 37'$ out of the Tabular Dip $68^{\circ} 22'$. The Remainder is $66^{\circ} 45'$, for the true Dip at *Boston* with that Needle. Thus if you want the true Dip, by the same Needle, at *Dronthem* in *Norway*, *Northward*: Because the equal Distance of *Dronthem* from the Magnetick Pole is 15, 2 Parts of 29, 6 the Distance of *London* from that Pole; deduct $12' = 62' = 1^{\circ} 2'$ out of the Tabular Dip $82^{\circ} 30'$, and the Remainder, $81^{\circ} 28'$ is the true Dip at *Dronthem*, with that Needle: And so in all other Cases whatsoever.

N. B. The Table before set down, supposes that the true Dip differs according to such a Line of Sines, whose middle Point gives 66° on one Side, and 24 on the other; and is made by adding or subtracting 8 to the Complement of the Dip found by the natural Sines for every $\frac{1}{2}$ of equal Distances from the Equator or Pole.

N. B. If any desire to calculate by Trigonometry the Distances of all Places from the magnetick Equator or Poles, and the Distances of that Equator and those Poles in every particular Case, both made use of in the foregoing Calculations, it is thus to be done:

In the (Fig. 12.) Triangle BLA we have BL the Co-Latitude of *London*; BA the Co-Latitude of the magnetick *North* Pole; and the included Angle, ABL = the Distance of the Meridian of that Pole from the Meridian of *London*; to find the Angle QAM and the Side AL . Then in

In the Triangle $\triangle AM$, we have the Angles $\angle AM$ and $\angle MA$; and the Side AM , = the Distance of the Magnetick Pole from the Magnetick Equator, to find AL . So we have the Proportion of AL to AM , $\angle E$ to $\angle A$.

But since the *Data* are not yet sufficiently exact for the Calculation, measuring is sufficient.

F I N I S.



Fig. 2.

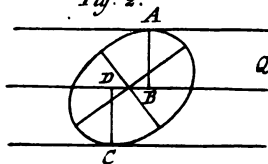


Fig. 3.

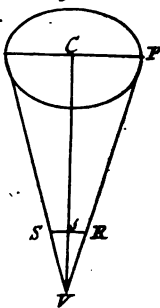


Fig. 4.

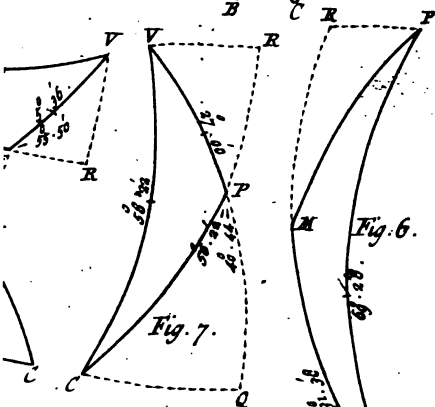
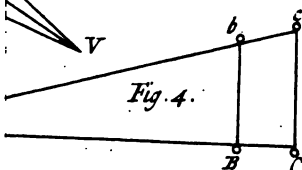


Fig. 6.

Fig. 7.

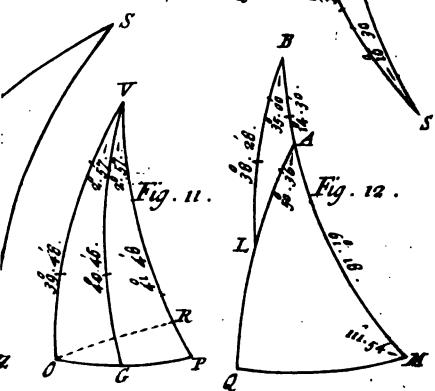


Fig. 11.

Fig. 12.

30
171 CH.

